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Model Predictive Control for constrained navigation of autonomous vehicles

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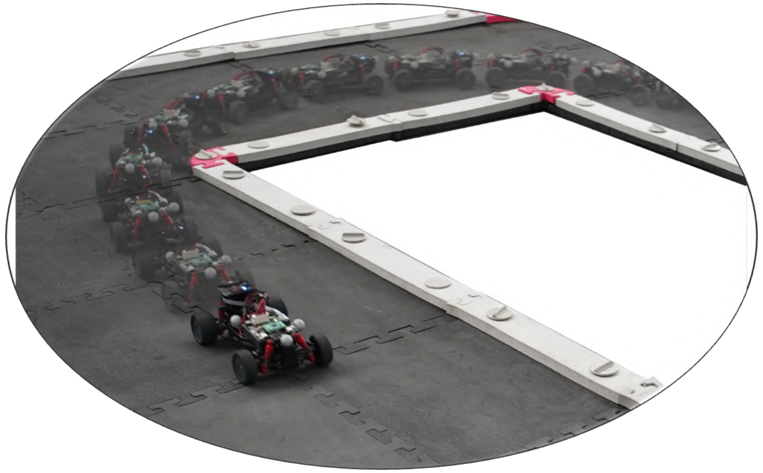
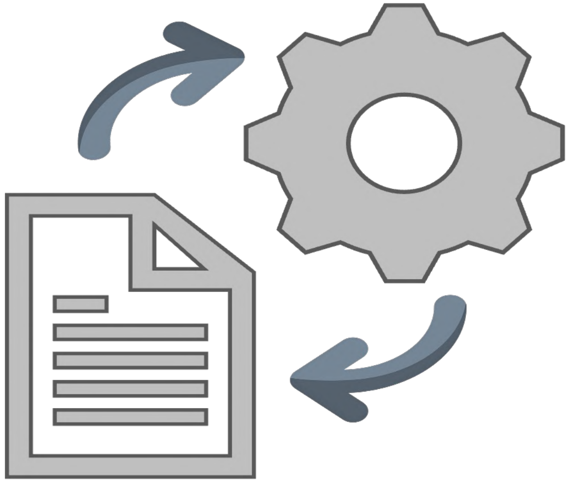
Practical applications



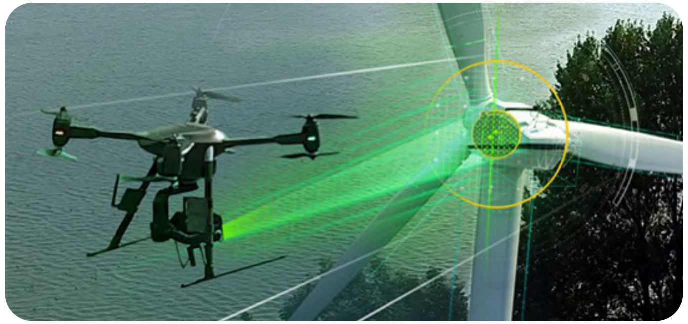


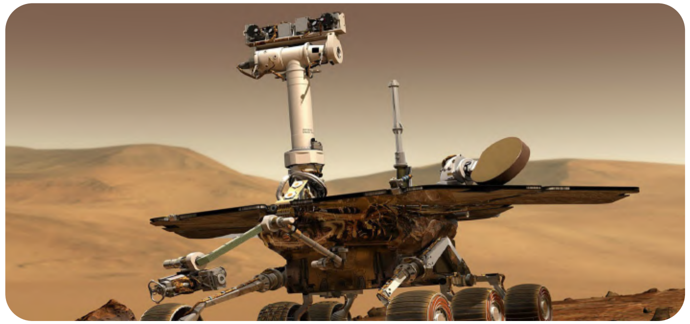
Theoretical findings

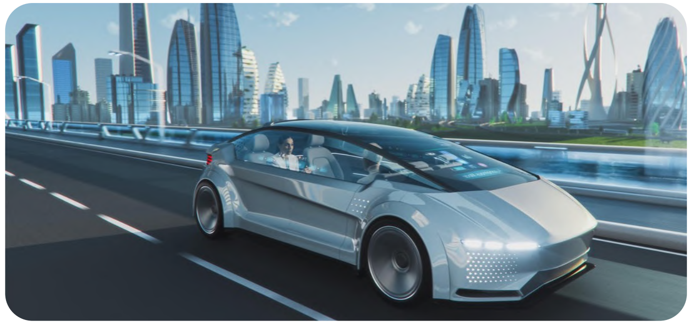
Motivation

Known environments

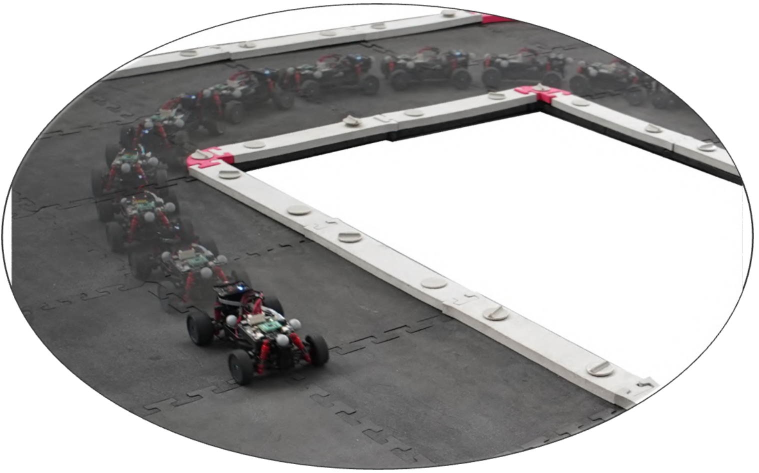
..highly heterogeneous and unknown environments



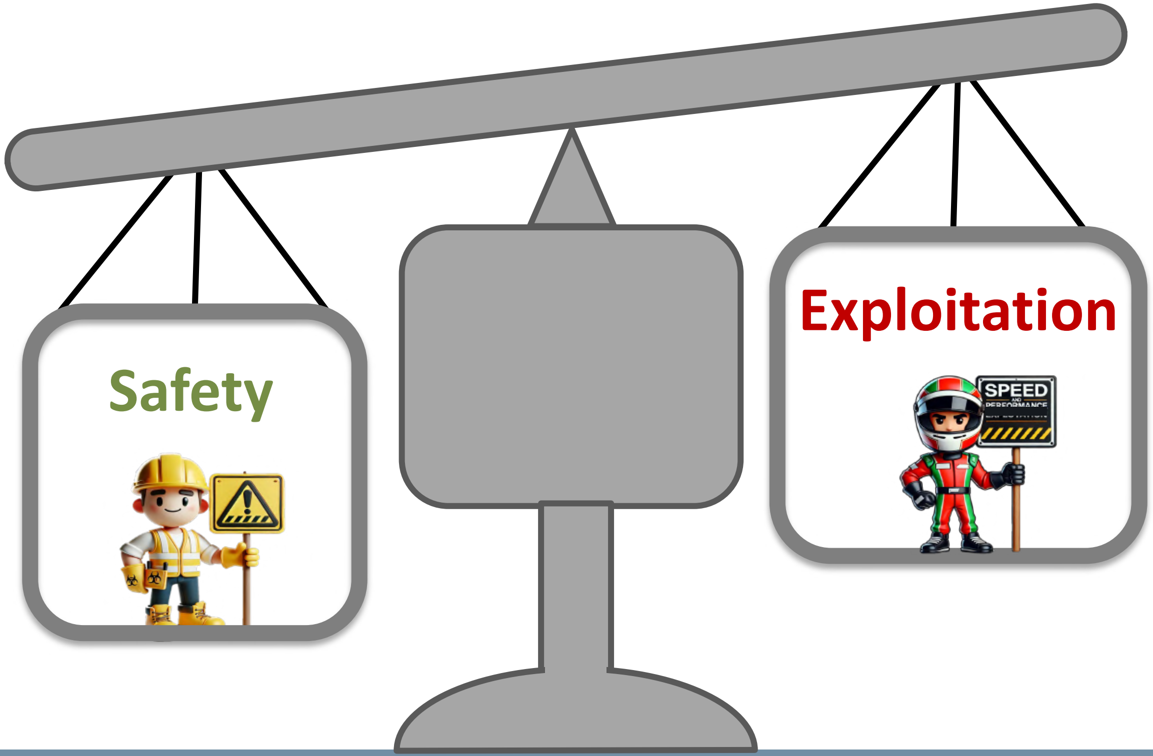




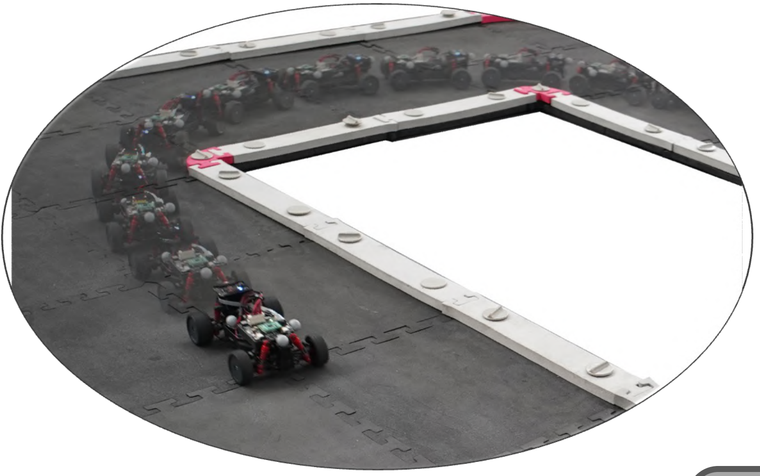
Motivation



“How can I safely navigate an unknown environment without colliding with the surrounding?”

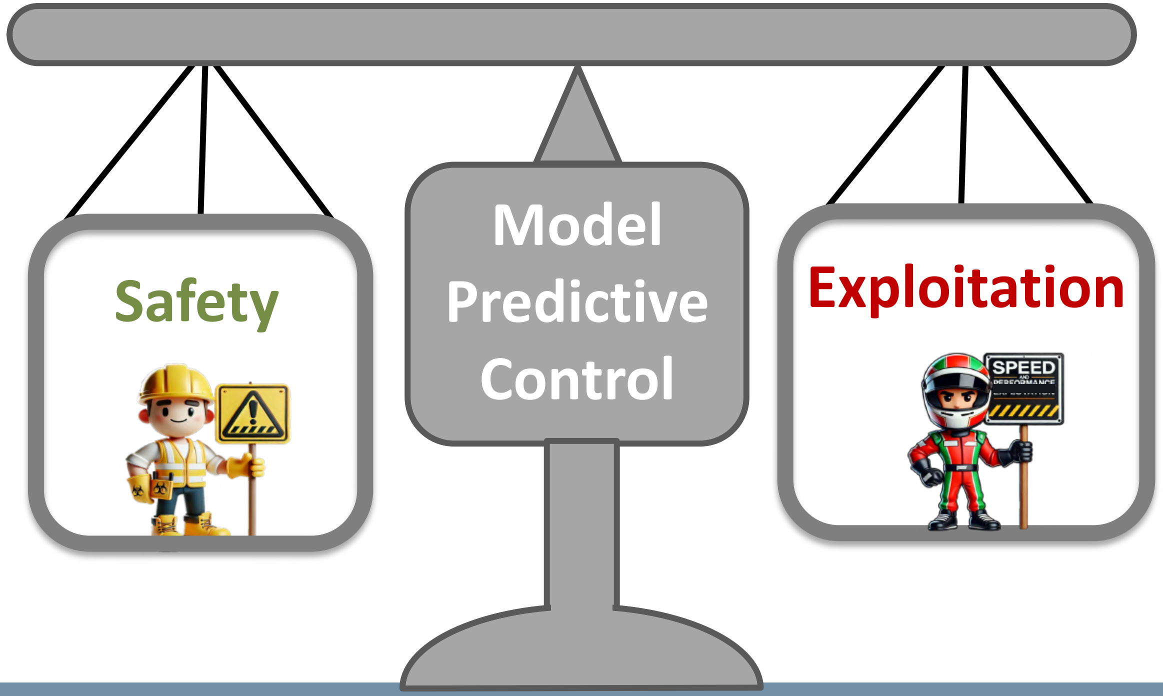


Motivation



"How can I safely navigate an unknown environment without colliding with the surrounding?"

Constraints satisfaction



Optimize performance (cost function minimization)

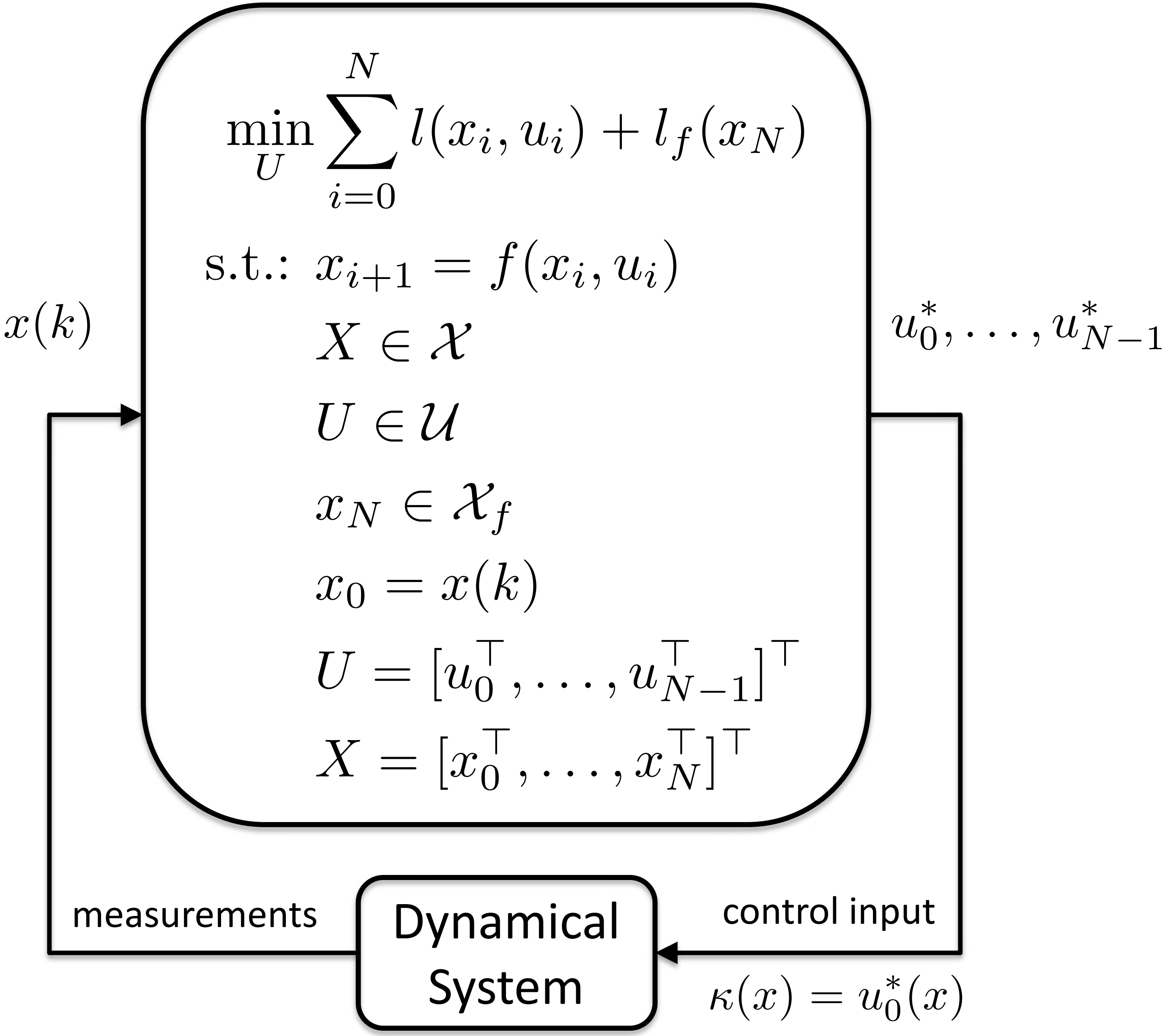
Model Predictive Control

RH Algorithm

i. At each **time k**: solve FHOCP.
ii. Apply to the system the first input in the optimal sequence
iii. At **time $k+1$**: Get new measurements and repeat the optimization.

Elements of the FHOCP

- Current state
- System model
- Terminal ingredients
  - Recursive feasibility
  - Constraint satisfaction
  - (Stability)
- Cost function
- Constraints



Model Predictive Control

Terminal ingredients

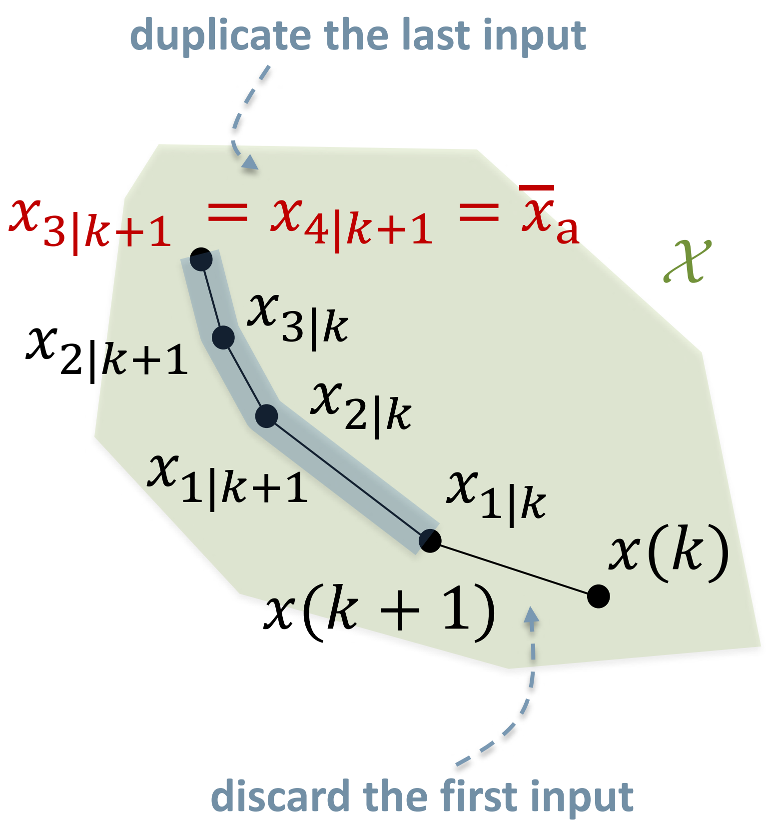
Terminal equality constraint

$$l_f = 0, \qquad \kappa_f = \bar{u}_a \qquad \mathcal{X}_f = \bar{x}_a$$

✓ Easy to satisfy when considering position-invariant systems (like vehicles)

Recursive feasibility

If the FHOCP admits a solution at time $k = 0$, then a solution to the MPC optimization problem exists $\forall k > 0$.

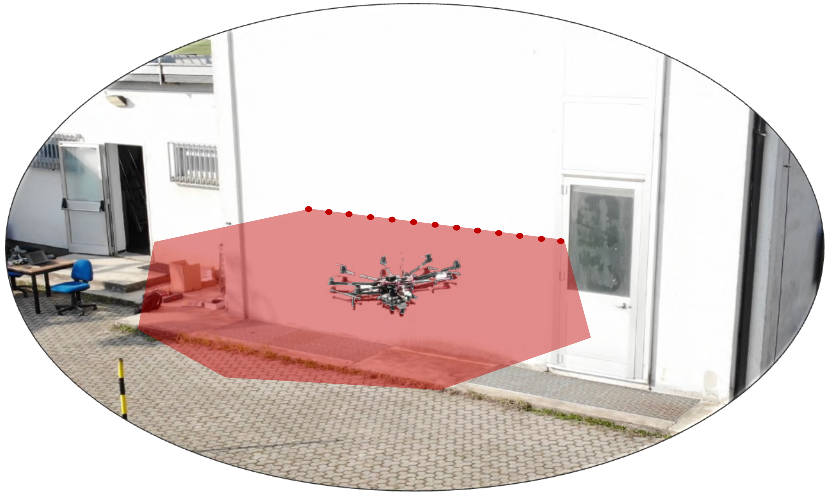


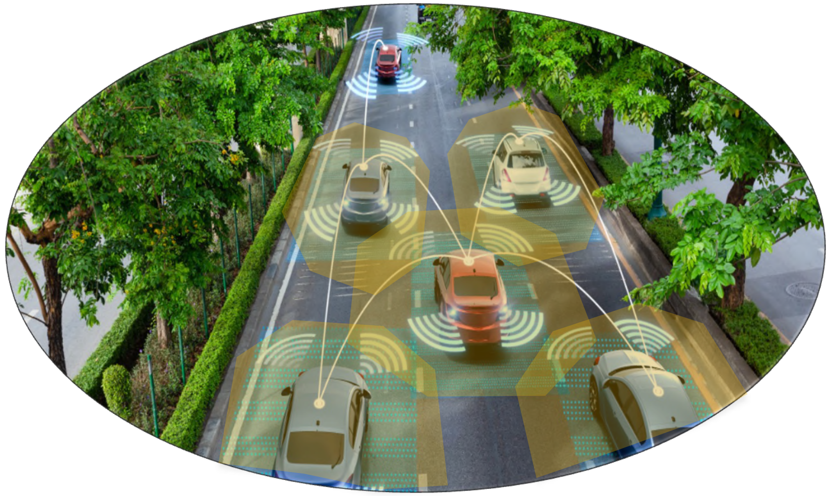
MPC for constrained navigation

State constraints from local sensors

- The system evolves in a partially **unknown environment** and the surrounding is detected by **onboard sensors** (LiDARs, cameras, antennas…).
- **Safe set** $\mathcal{X}(\mathbf{k}, \mathbf{x}(\mathbf{k}))$ around the vehicle position (state)

time-varying state constraints $x_i \in \mathcal{X}(k, x(k)) \qquad \forall i \in \mathbb{N}_0^N$





time-varying constraints can lead to a loss of feasibility!

MPC for constrained navigation

Safety - Shifting state constraints

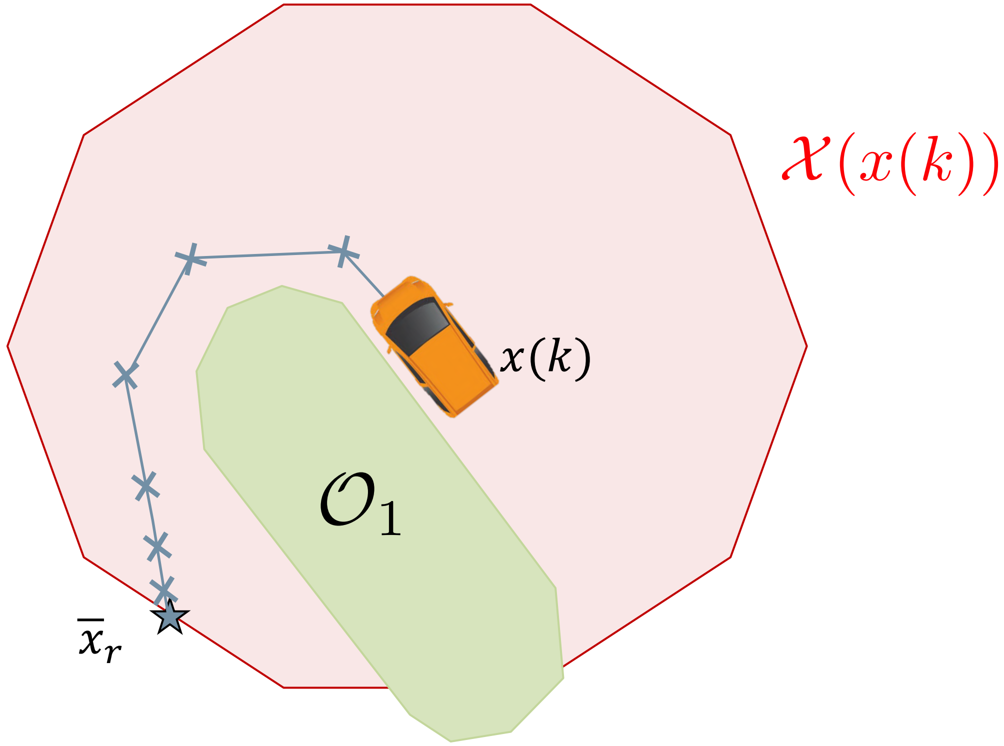
$$\mathcal{X}(x(k)) = x(k) \oplus \mathcal{H}$$

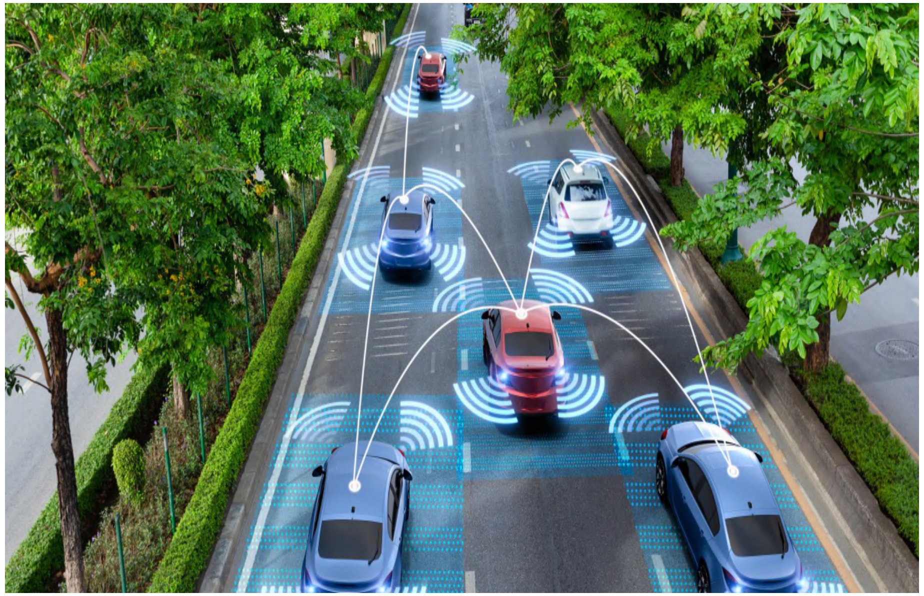
$\mathcal{H}$: convex closed polytopic sets containing the origin

$x(k)$: solution at time $\boldsymbol{k}$

$$\mathcal{X}(x(k)) = \{\xi \in \mathbb{R}^n : H(\xi - x(k)) \leq h\}$$

H, h: time-invariant





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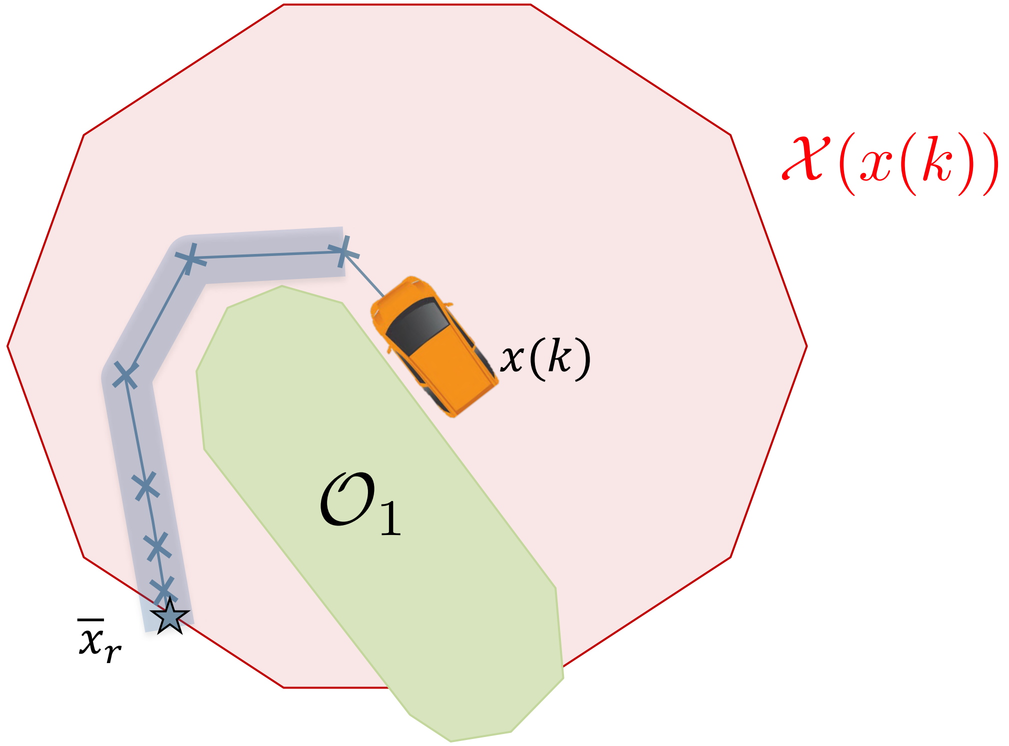
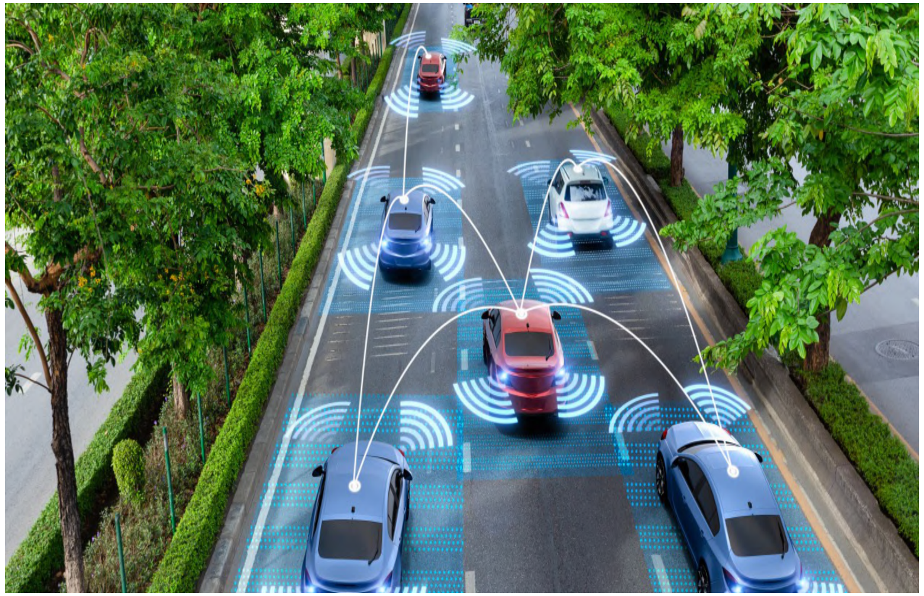
time-invariant

Candidate solution at time $\boldsymbol{k+1}$





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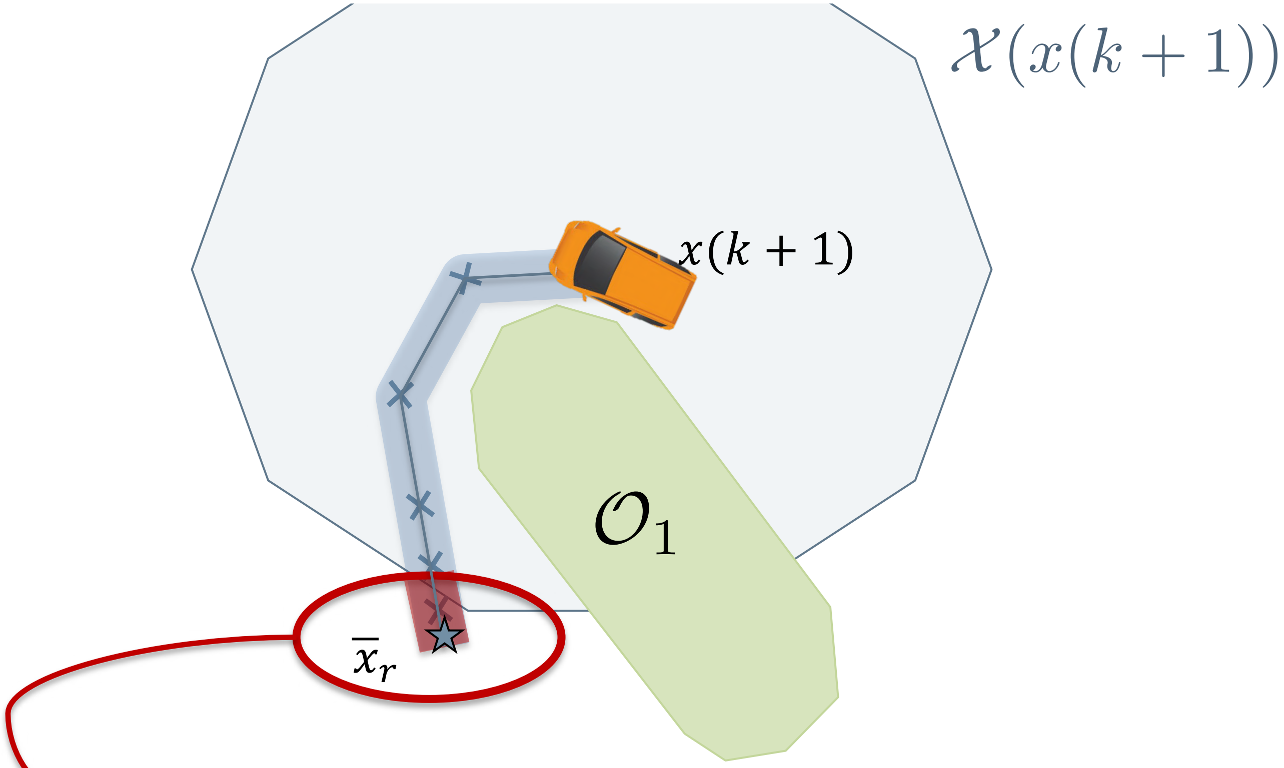
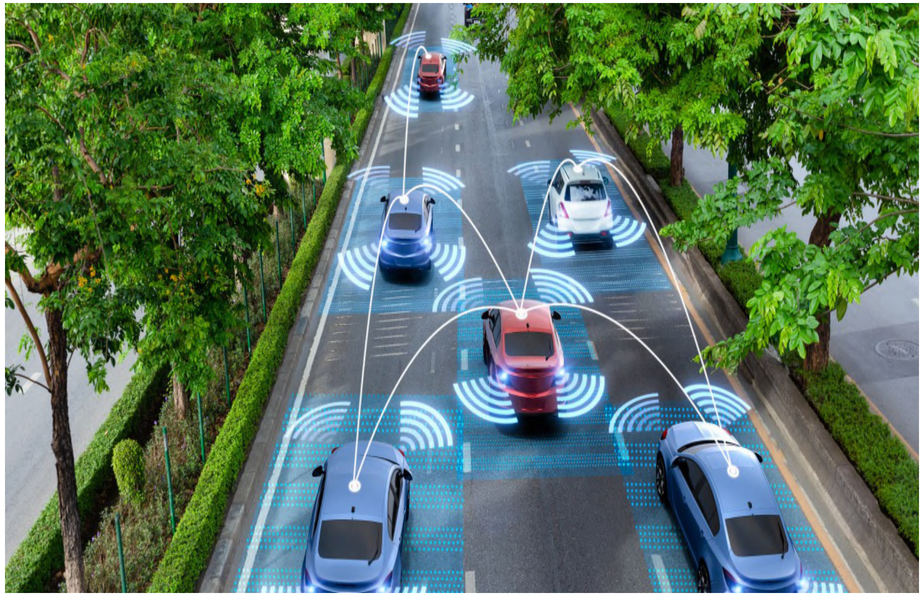


The candidate solution violates the constraint at time $k+1$



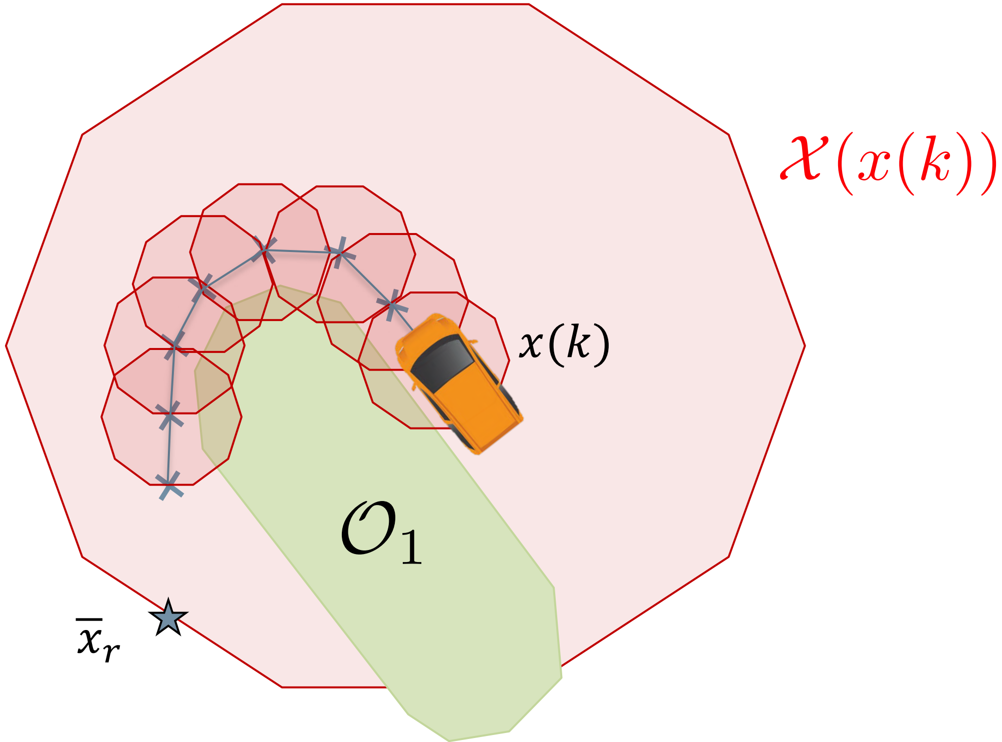
MPC for constrained navigation

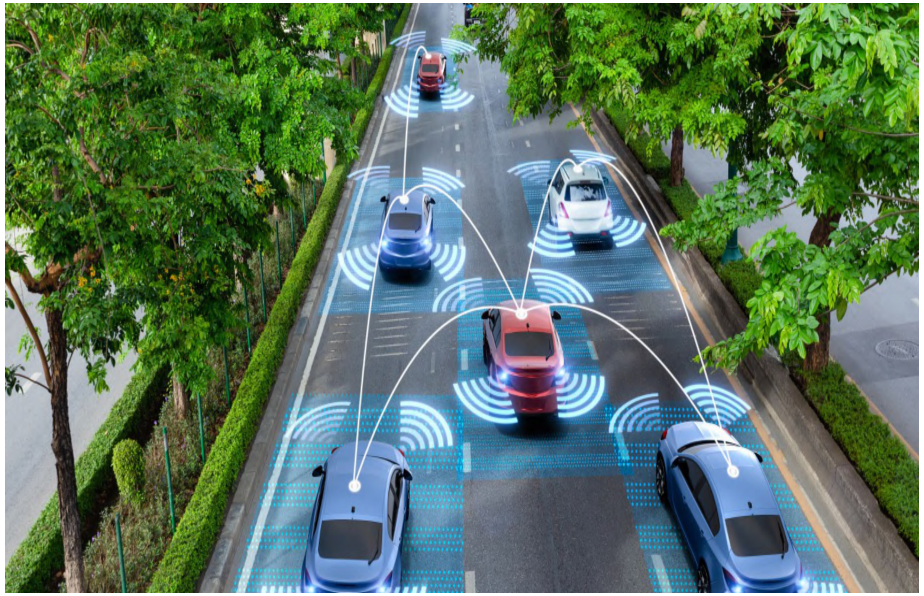
Safety - Shifting state constraints

✓ The following implementation guarantee the **recursive feasibility** of the problem

$$H(x_{i+1} - x_i) \leq h_i \quad \text{such that} \quad \sum_{i=0}^{N} h_i \leq h, \qquad \forall i \in \mathbb{N}_0^{N-1}$$

solution at time ***k***





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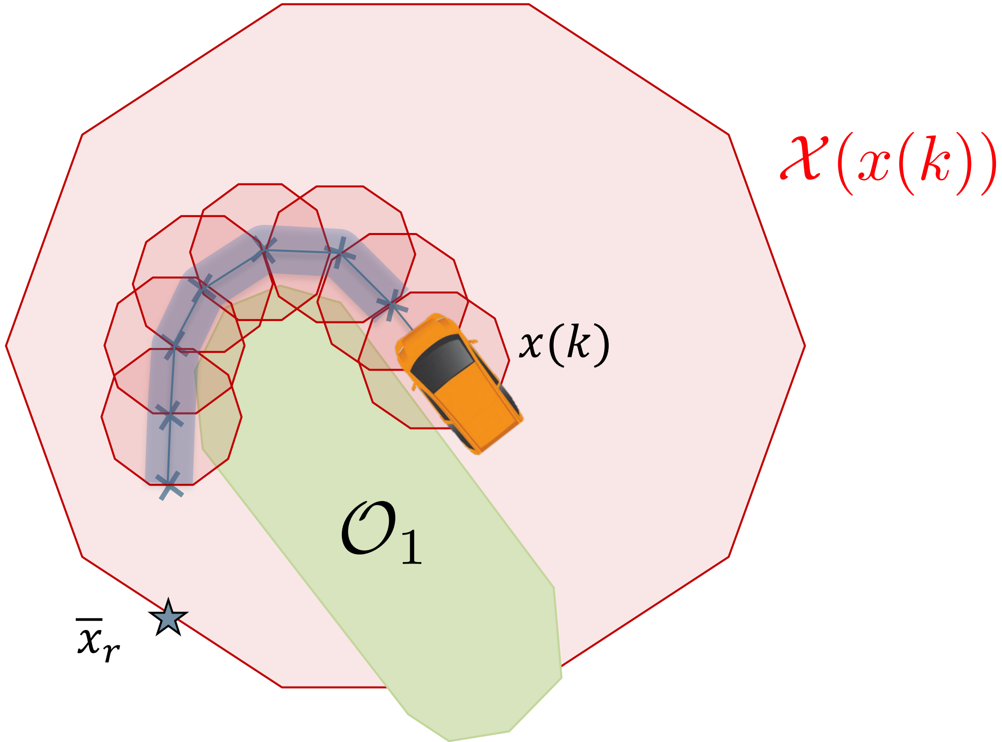
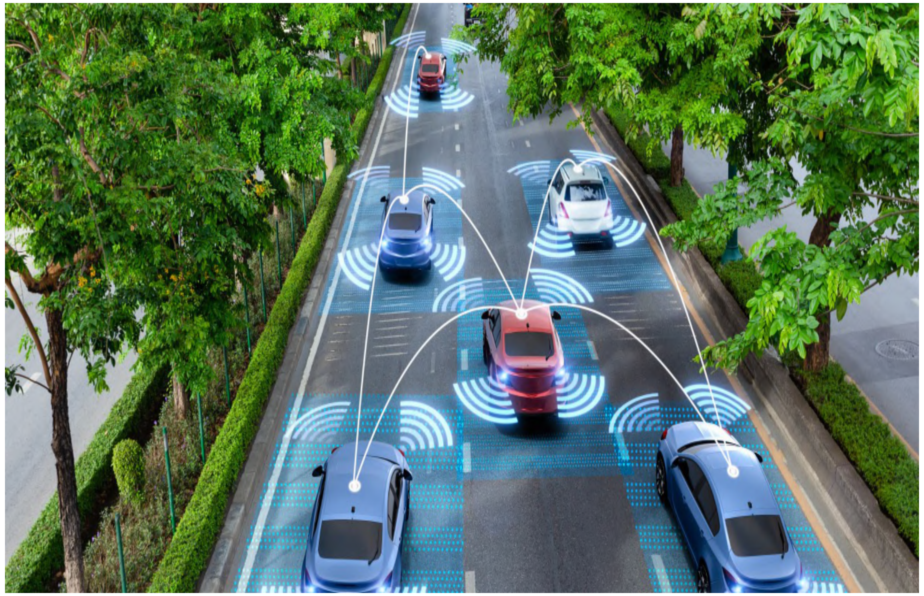
Candidate solution at time $\boldsymbol{k+1}$





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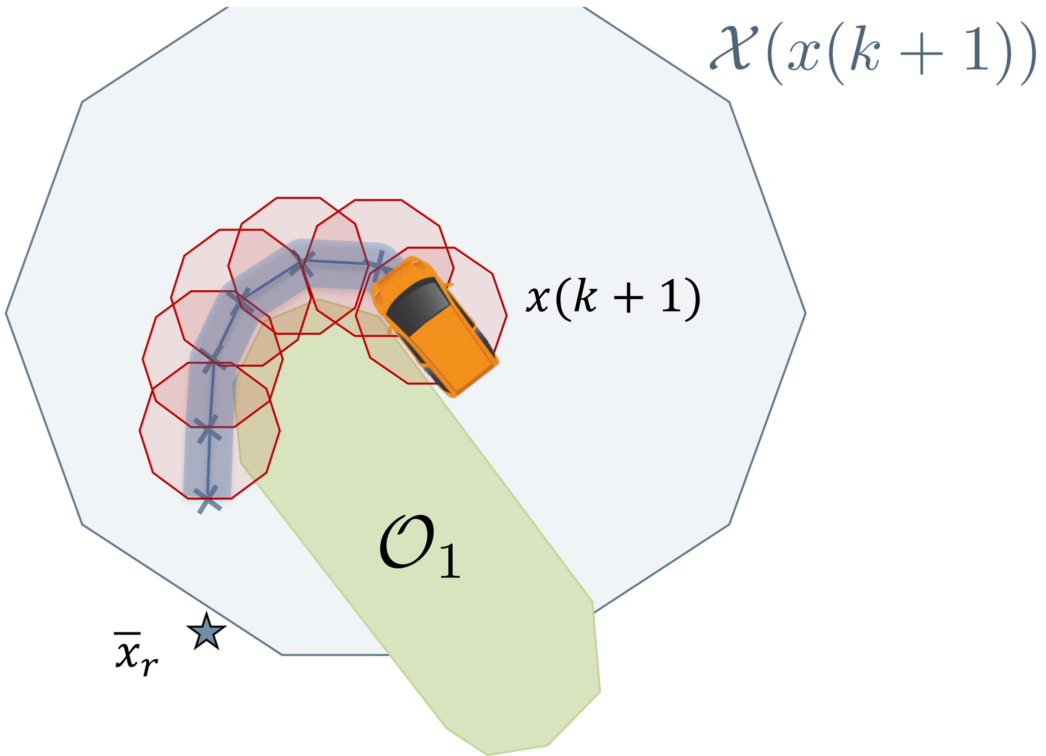
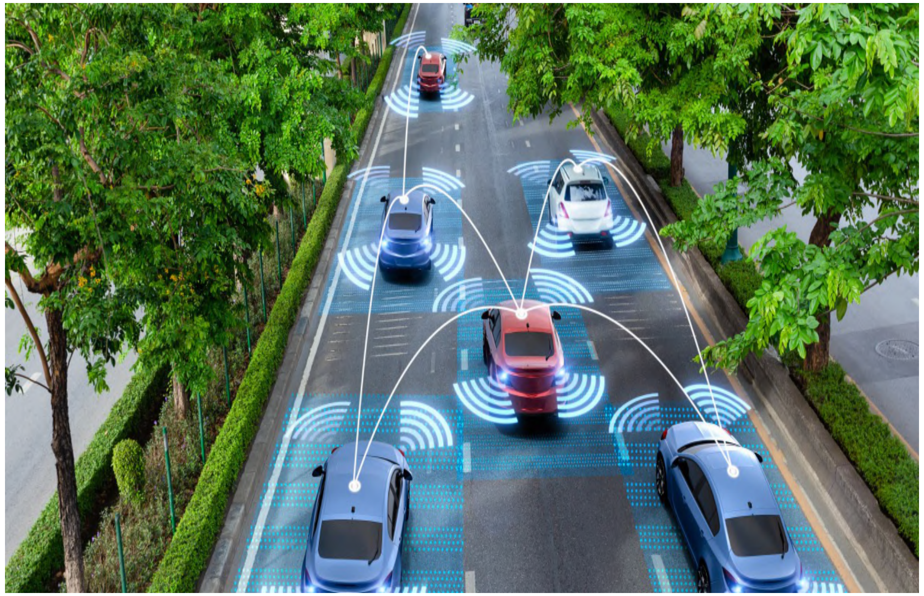




MPC for constrained navigation

Safety - Unpredictable state constraints

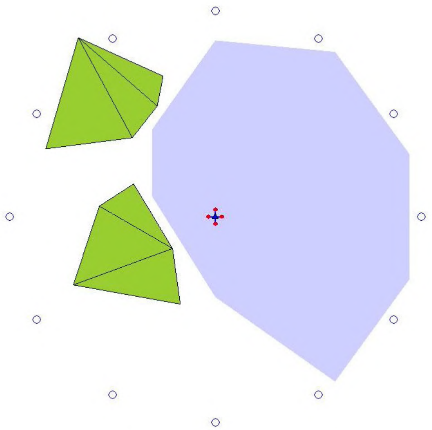
$$\mathcal{X}(k, x(k)) = x(k) \oplus \mathcal{H}(k)$$

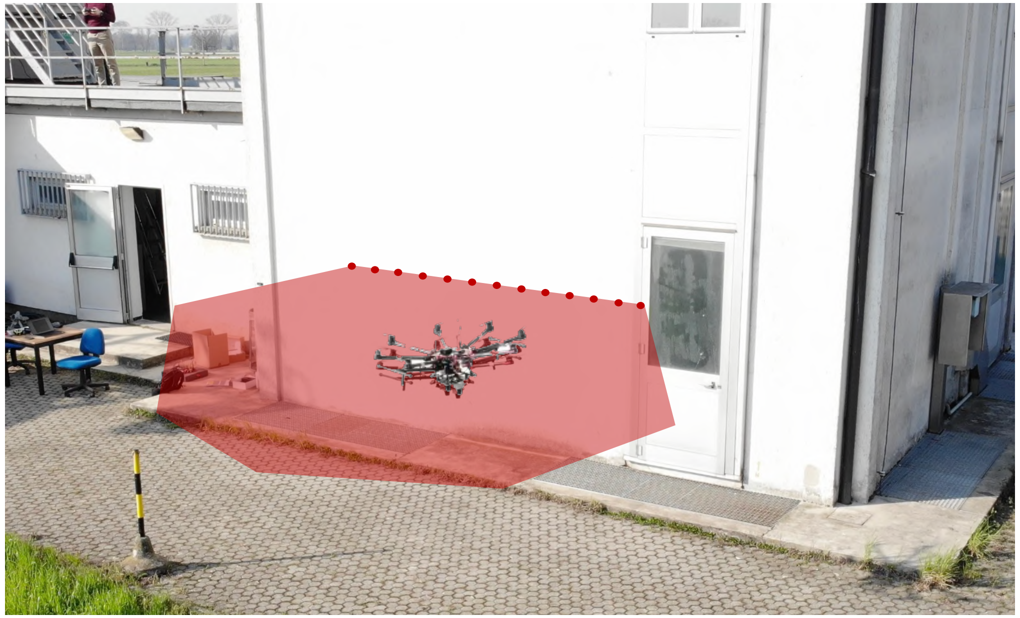
convex closed polytopic sets containing the origin

$$\mathcal{X}(k, x(k)) = \{\xi \in \mathbb{R}^n : H(k)(\xi - x(k)) \leq h(k)\}$$

time-variant

Due to the time-varying unpredictable nature of the constraints the **recursive feasibilty cannot be guaranteed**.





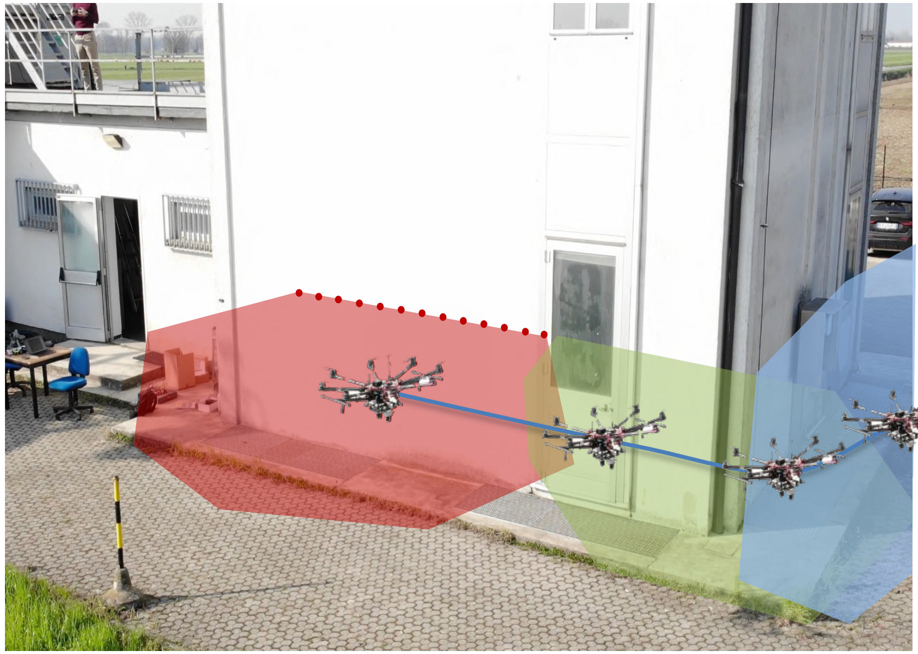
[2] Saccani, D., & Fagiano, L. Autonomous uav navigation in an unknown environment via multi-trajectory model predictive control. In 2021 European Control Conference (ECC)

MPC for constrained navigation

Safety - Unpredictable state constraints

Sequence of feasible sets

$$\mathcal{S}(k) = \{\mathcal{X}(j, x(j)), \forall j \leq k\}, \qquad \forall k$$



Admissible state

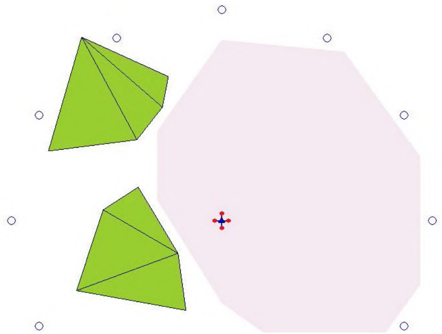
$$x(k) \in \mathcal{S}(k),$$

$$\text{if exists a } j \leq k \text{ such that } x(k) \in \mathcal{X}(j, x(j)).$$

Modified problem

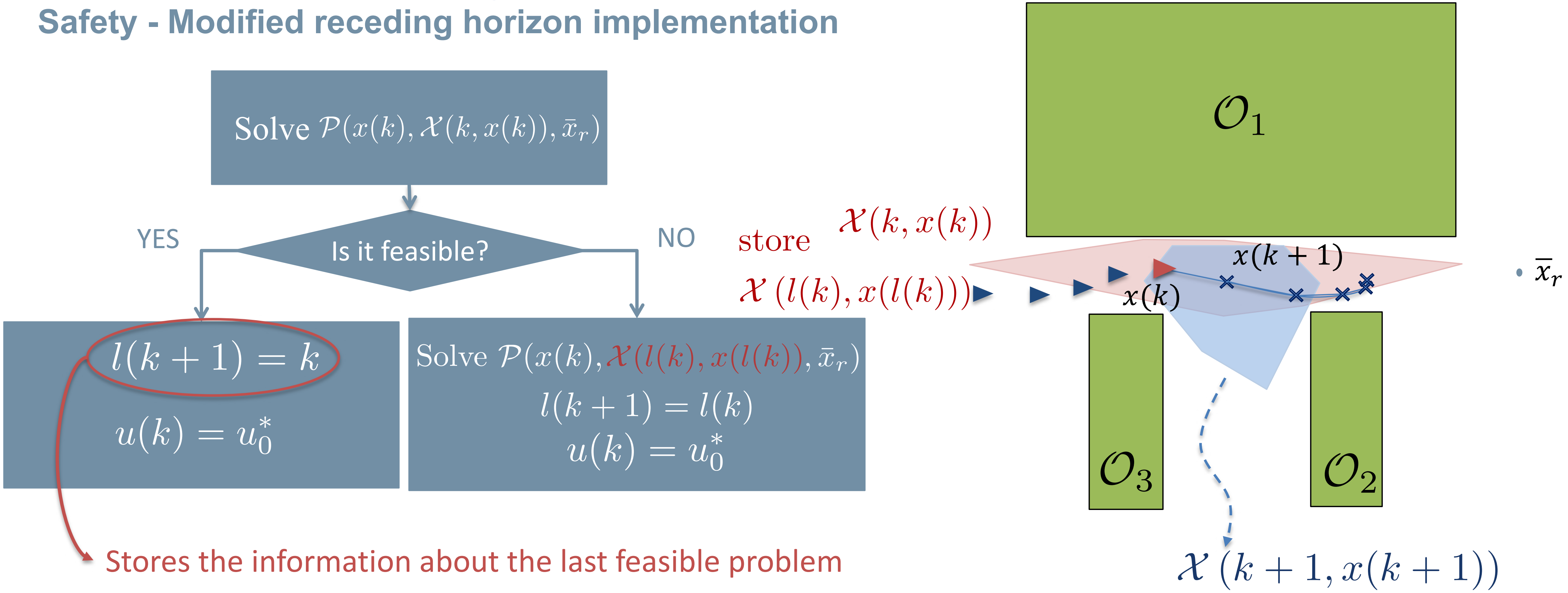
Instead of: $x_i \in \mathcal{X}(k, x(k))$

Guarantee the existence of a feasible problem at each time step, such that: $x(k) \in \mathcal{S}(k)$



MPC for constrained navigation

Safety - Modified receding horizon implementation



✓ Easy to prove the existence of an admissible control problem at each time step.

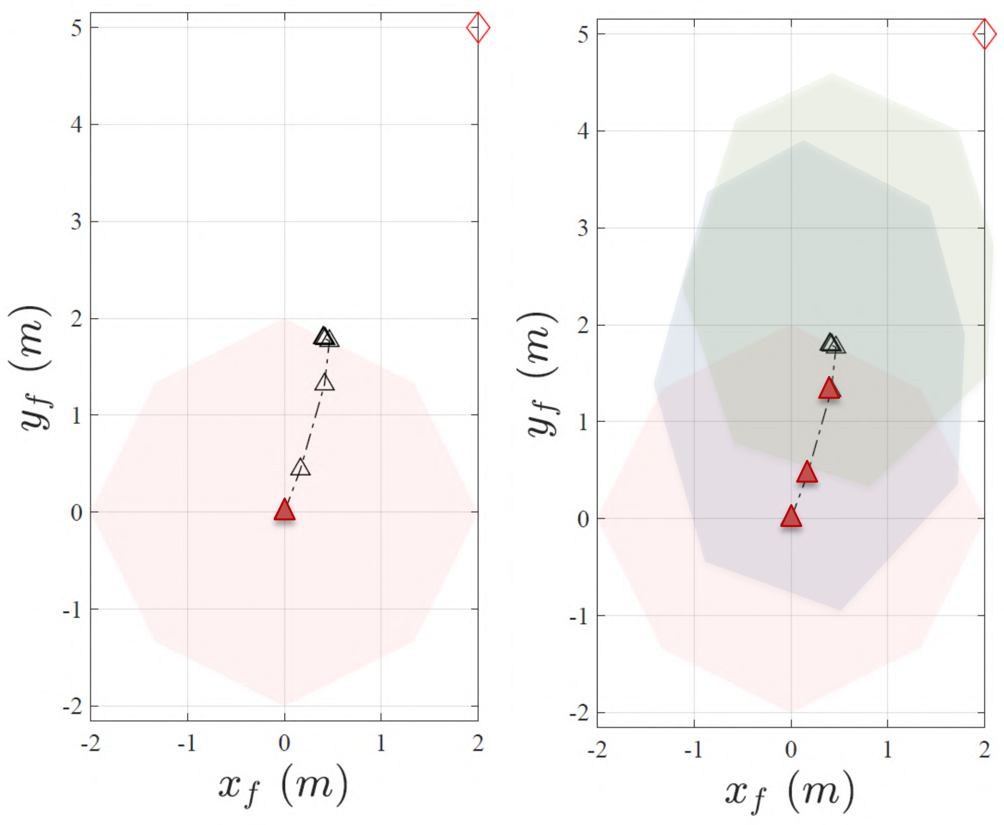
[2] Saccani, D., & Fagiano, L. Autonomous uav navigation in an unknown environment via multi-trajectory model predictive control. In 2021 European Control Conference (ECC)

MPC for constrained navigation
Exploitation

Safe set $\mathcal{X}(\mathbf{k})$ considered at each time step k **generally changes over time**

- Relying only on local information can lead to a **conservative behaviour.**

Multi-trajectory MPC formulation

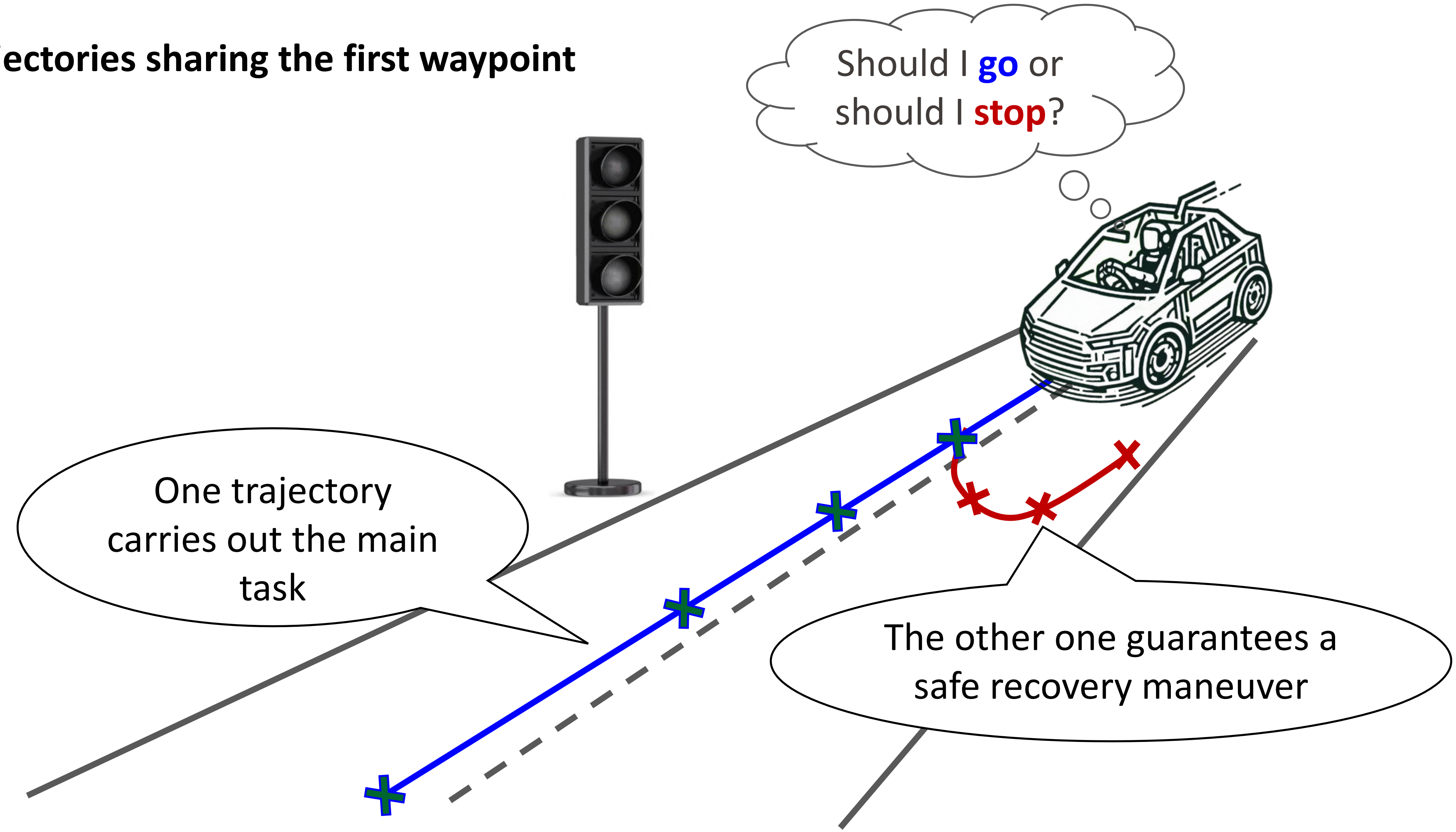


[3] Saccani, D., Cecchin, L., & Fagiano, L. (2022). Multitrajectory model predictive control for safe UAV navigation in an unknown environment. IEEE Transactions on Control Systems Technology

Multi-trajectory Model Predictive Control (mt-MPC)

Intuitive idea

- **Predict two trajectories sharing the first waypoint**



Multi-trajectory Model Predictive Control (mt-MPC)

Formulation

$x(k)$

- Two different input sequences sharing the first control action $\mathbf{u_0^e} = \mathbf{u_0^s}$

$$U^e = [\mathbf{u_0^e}^\top, \dots, {u_{N-1}^e}^\top]^\top \qquad U^s = [\mathbf{u_0^s}^\top, \dots, {u_{N-1}^s}^\top]^\top$$

- Predict two different state trajectories sharing the first state $\mathbf{x_1^e} = \mathbf{x_1^s}$

$$X^e(x(k), U^e) = [\mathbf{x_1^e}^\top, \dots, {x_N^e}^\top]^\top \qquad X^s(x(k), U^s) = [\mathbf{x_1^s}^\top, \dots, {x_N^s}^\top]^\top$$

Exploitation trajectory ignores the time-varying constraints, but it is considered in the cost

Safe trajectory is used to satisfy time-varying constraints

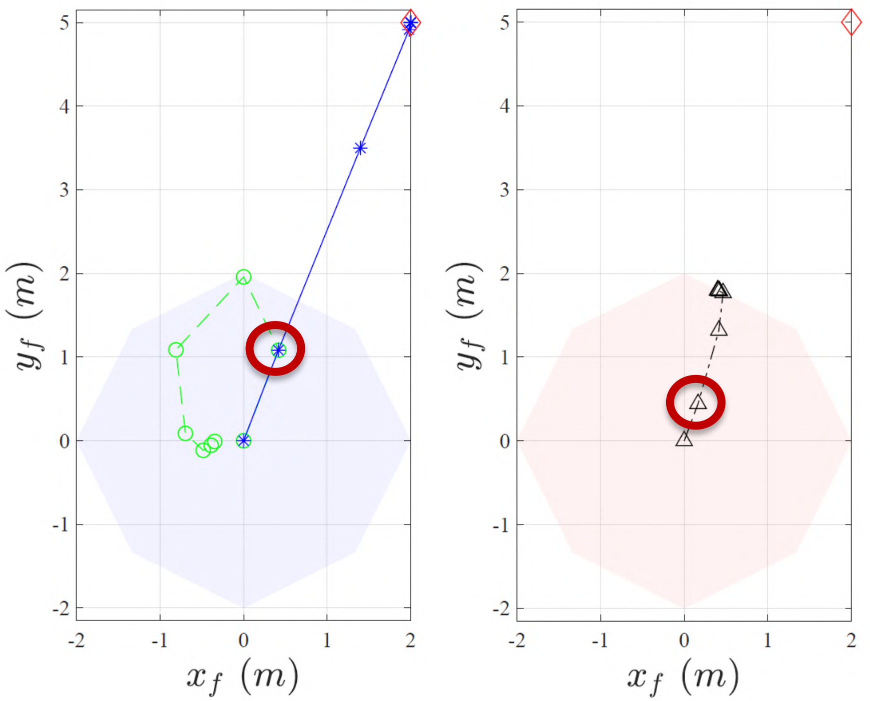
$$\min_{U^{e,s}} \sum_{i=0}^{N} l^e(x_i^e, u_i^e)$$

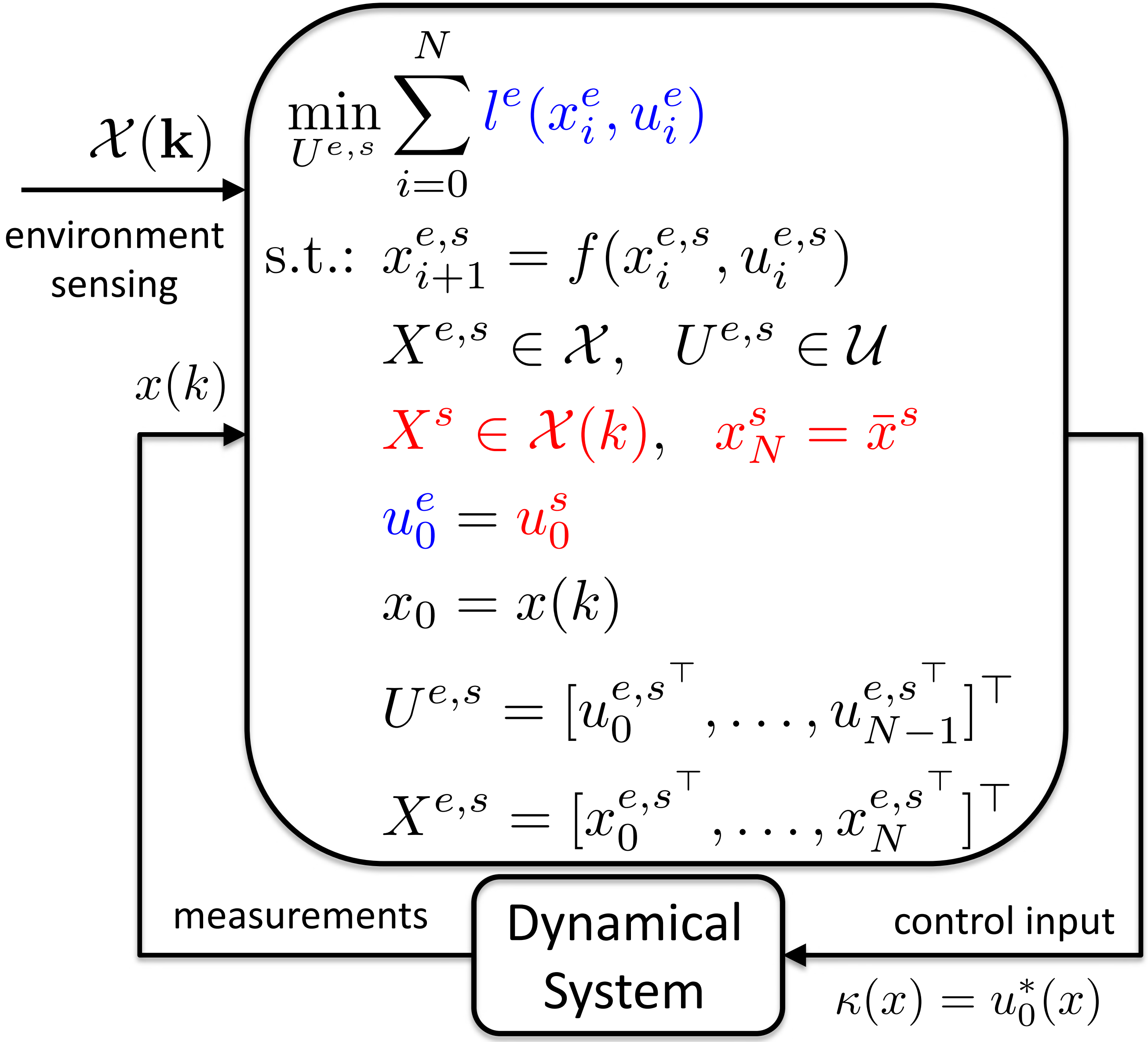
$$X^s \in \mathcal{X}(k), \quad x_N^s = \bar{x}^s$$

Multi-trajectory Model Predictive Control (mt-MPC)

Formulation

- Partially decouple constraint satisfaction (safety) from cost function minimization (exploitation)





mt-MPC for safe UAV navigation in an unknown environment

- Navigate a commercial drone in an unknown static environment
- Guarantee persistent obstacle avoidance despite the various sources of uncertainty (disturbances, model plant mismatch,…)
- Use only real time LiDAR measurements

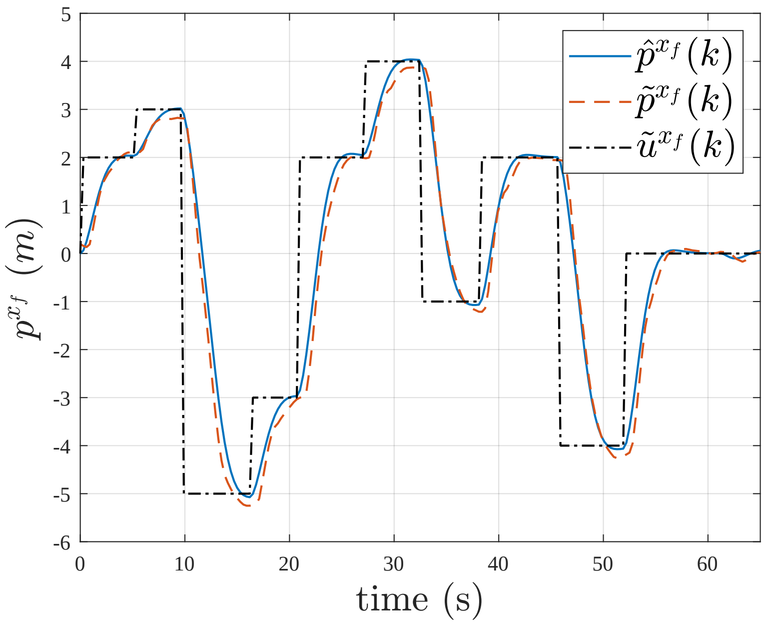
[3] **Saccani**, D., Cecchin, L., & Fagiano, L. (2022). Multitrajectory model predictive control for safe UAV navigation in an unknown environment. IEEE Transactions on Control Systems Technology

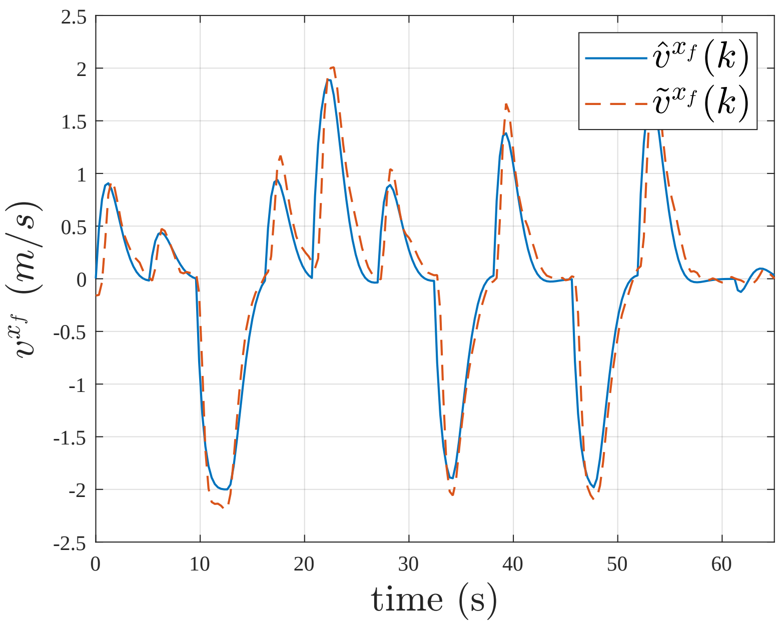
[4] Cecchin, L., **Saccani**, D., & Fagiano, L. (2021). G-beam: Graph-based exploration and mapping for autonomous vehicles. In 2021 IEEE Conference on Control Technology and Applications (CCTA)

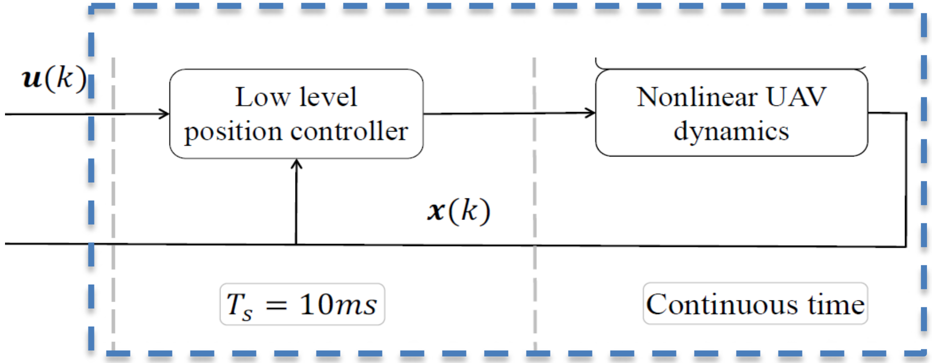
mt-MPC for safe UAV navigation in an unknown environment

Model - sensor - uncertainty

- The controlled drone behaviour, can be approximated with a **linear model** with state $x(k) = [p(k)\ v(k)]^\top$.







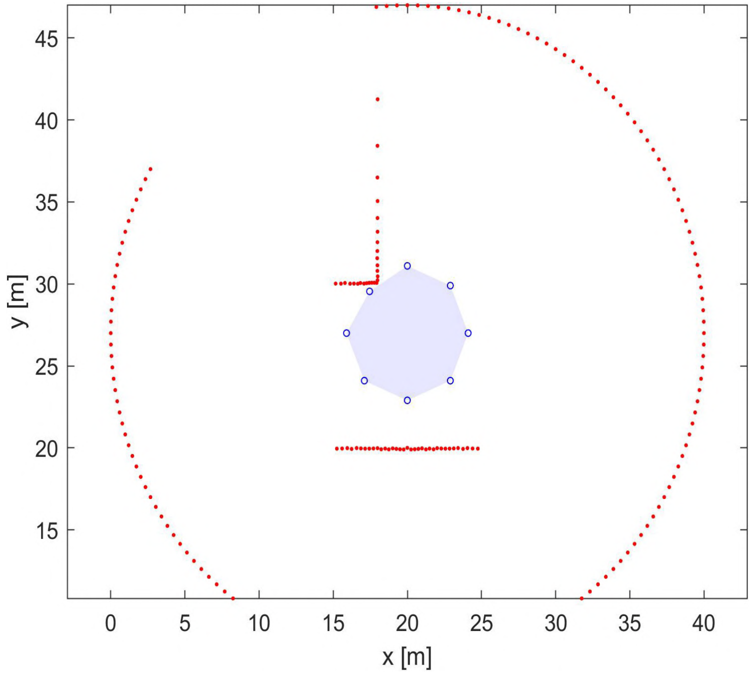
mt-MPC for safe UAV navigation in an unknown environment

Model - sensor - uncertainty

- The controlled drone behaviour, can be approximated with a **linear model** with state $x(k) = [p(k)\ v(k)]^\top$.

- The LiDAR measurements are used to derive an under-approximation of the free space with a **convex polytope.**

$$\mathcal{X}(k, x(k)) = \{\xi \in \mathbb{R}^n : H(k)(\xi - x(k)) \leq h(k)\}$$



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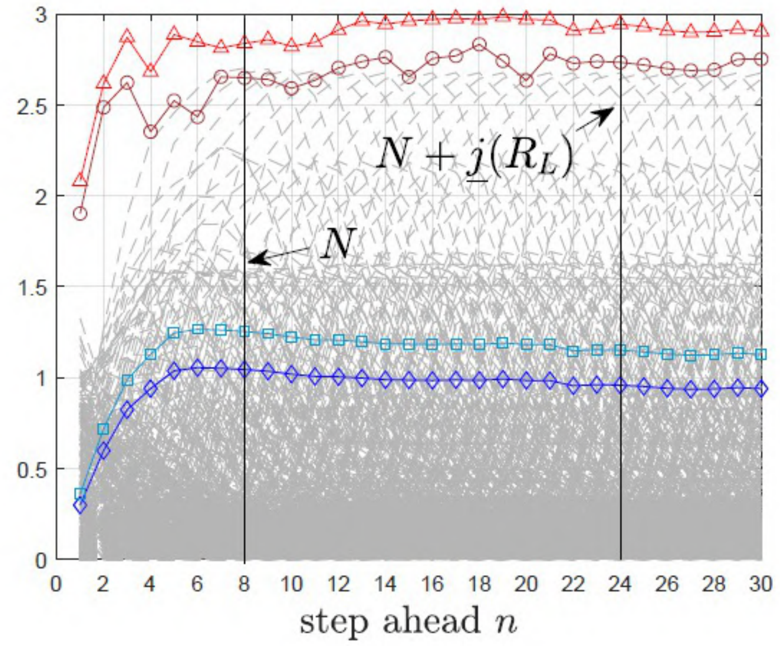
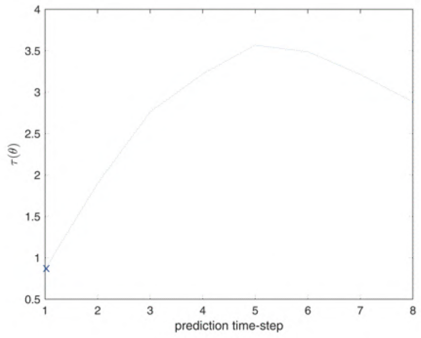
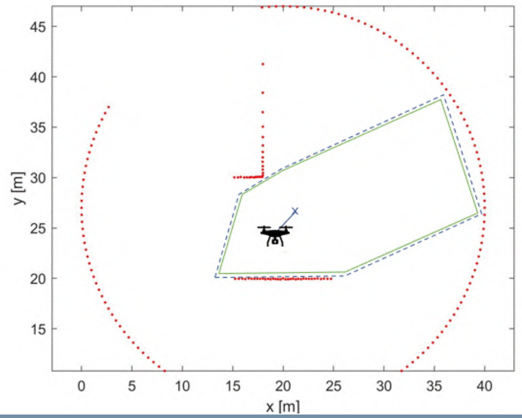
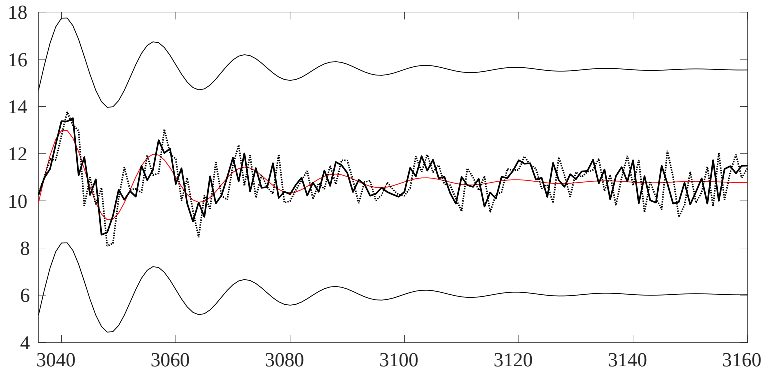
- The controlled drone behaviour, can be approximated with a **linear model** with state $x(k) = [p(k)\ v(k)]^\top$.
- The LiDAR measurements are used to derive an under-approximation of the free space with a **convex polytope.**
- Bounds on the prediction from data using a set membership approach to enhance the **robustness** of the MPC scheme.







constraints tightening ←



mt-MPC for safe UAV navigation in an unknown environment

Experimental results

Mt-MPC formulation

$$
\begin{aligned}
&\min_{U^{e,s}} \sum_{i=0}^{N} l^e(x_i^e, u_i^e) \\
\text{s.t.:}\ & x_{i+1}^{e,s} = f(x_i^{e,s}, u_i^{e,s}) \\
& X^{e,s} \in \mathcal{X}, \quad U^{e,s} \in \mathcal{U} \\
& X^s \in \mathcal{X}(k), \quad x_N^s = \bar{x}^s \\
& u_0^e = u_0^s \\
& x_0 = x(k) \\
& U^{e,s} = [u_0^{e,s\top}, \dots, u_{N-1}^{e,s\top}]^\top \\
& X^{e,s} = [x_0^{e,s\top}, \dots, x_N^{e,s\top}]^\top
\end{aligned}
$$

OSQP

$$
\min_x \frac{1}{2} x^T P x + q^T x
$$

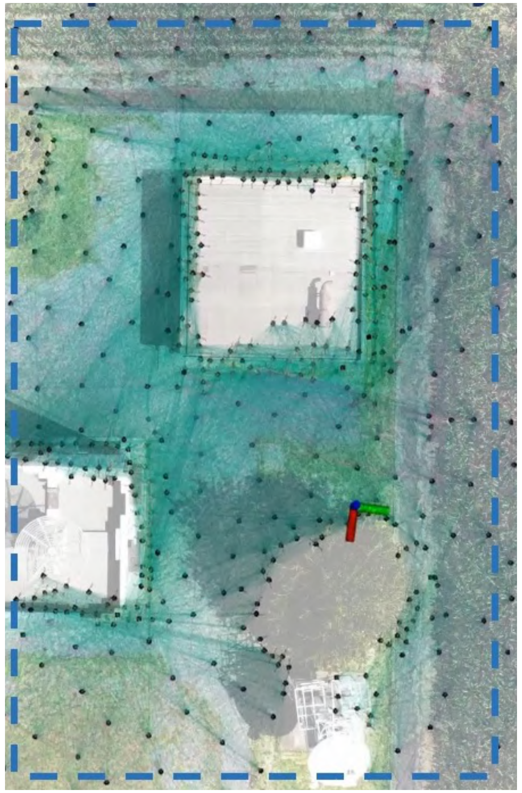
$s.t.:$

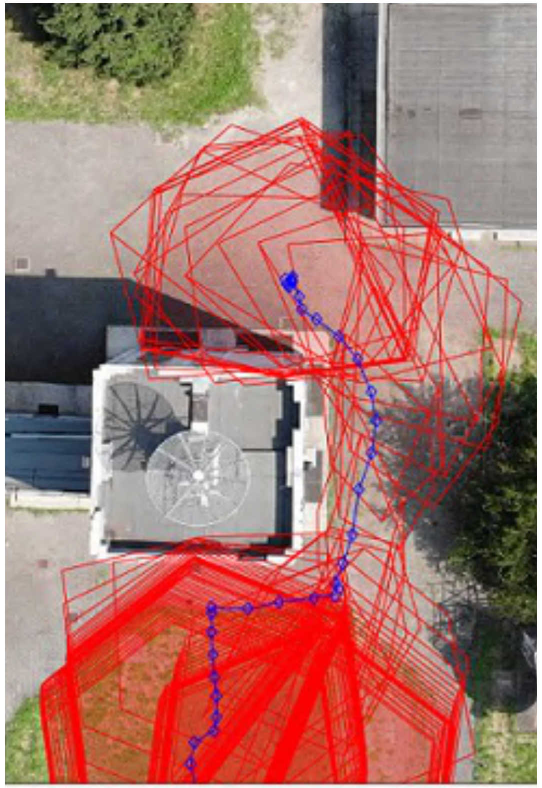
$$
\begin{aligned}
Gx &\leq h \\
Ax &= b \\
lb \leq x &\leq ub
\end{aligned}
$$

ROS

Robot Operating System

Onboard Computer





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mt-MPC for the navigation of multi-agent systems under limited communication

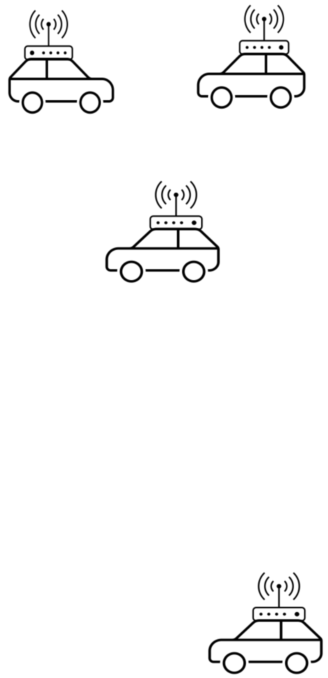
- Swarm of vehicles
- Communication between spatially close vehicles
- Guarantee persistent obstacle avoidance

[1] Saccani D., et al. Model predictive control for multi-agent systems under limited communication and time-varying network topology. In: 2023 IEEE Conference on Decision and Control (CDC).

mt-MPC for the navigation of multi-agent systems under limited communication

Model - communication - problem setup

- Swarm of M **vehicles** with nonlinear dynamics

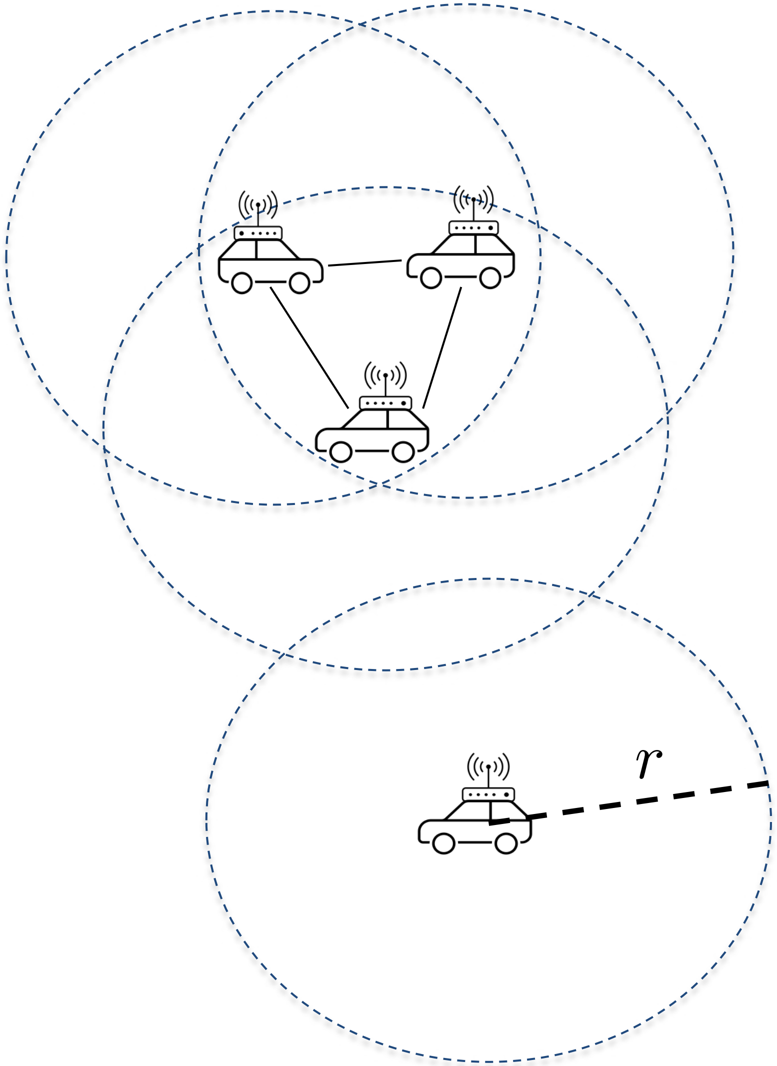


mt-MPC for the navigation of multi-agent systems under limited communication

Model - communication - problem setup

- Swarm of M **vehicles** with nonlinear dynamics
- **Communication** device
  - ➢ Spatially close vehicles can communicate

$$\|p_i(k) - p_j(k)\| \leq r$$



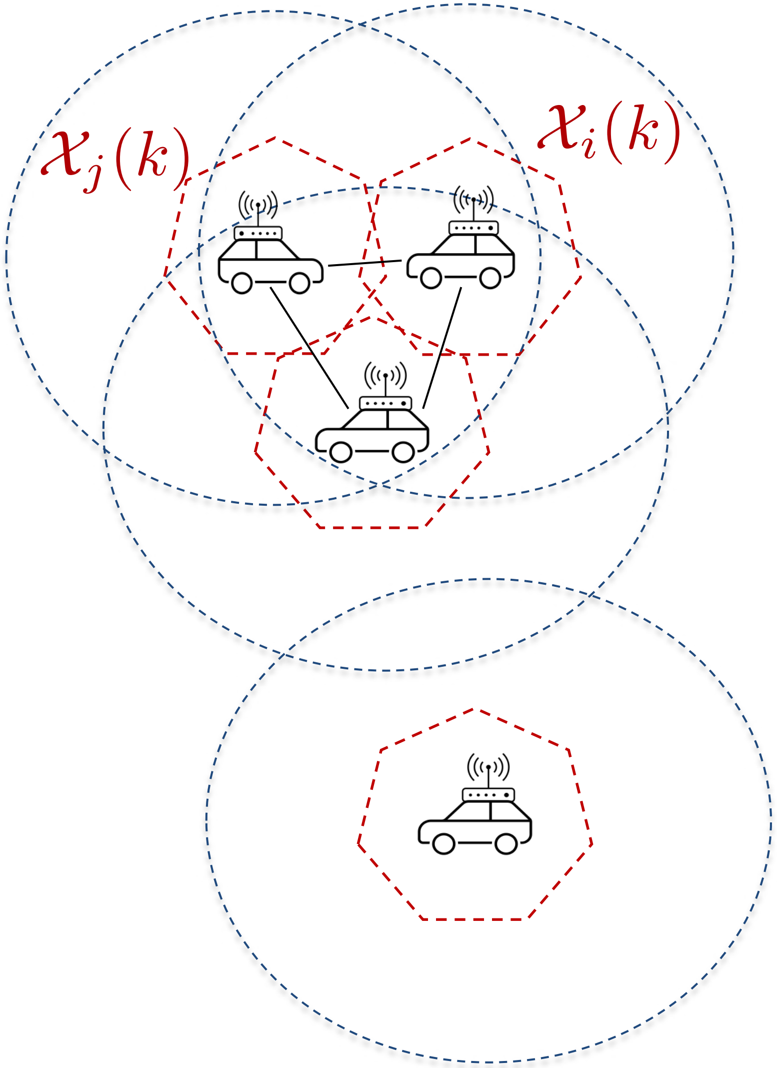
Model - communication - problem setup

- Swarm of M **vehicles** with nonlinear dynamics
- **Communication** device
  - Spatially close vehicles can communicate

$$\|p_i(k) - p_j(k)\| \leq r$$

  - Define a shifting safe set $\mathcal{X}_i(k)$ such that

$$\mathcal{X}_i(k) \cap \mathcal{X}_j(k) \neq \emptyset \implies \|p_i(k) - p_j(k)\| \leq r$$



mt-MPC for the navigation of multi-agent systems under limited communication

Model - communication - problem setup

- Swarm of $\boldsymbol{M}$ **vehicles** with nonlinear dynamics
- **Communication** device
  - Spatially close vehicles can communicate

$$\|p_i(k) - p_j(k)\| \leq r$$

  - Define a shifting safe set $\mathcal{X}_i(k)$ such that

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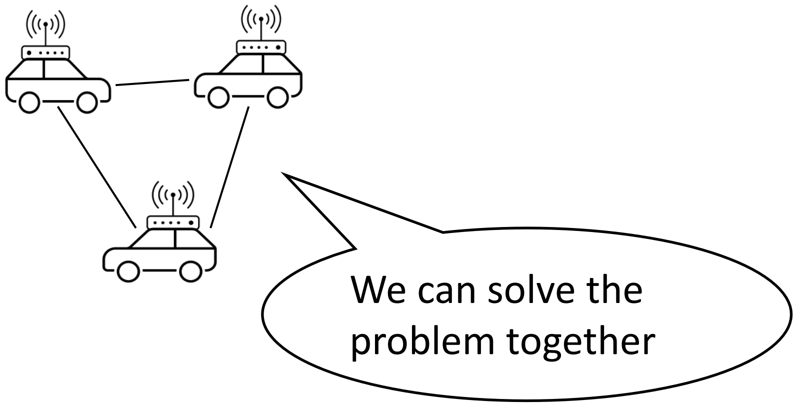
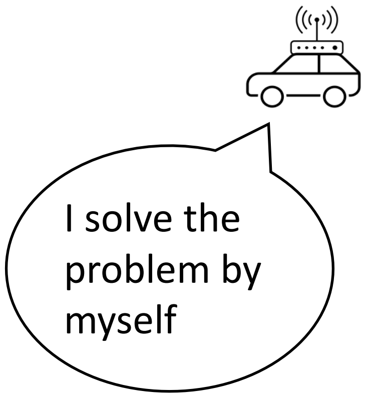
- Agents that can communicate will solve the problem together





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- Agents that can communicate will solve the problem together
- The **communication graph** is **time-varying** and position dependent
  - **Plug-in/out operations** of other vehicles are allowed without request

Conclusions

- **MPC** is a promising approach for the constrained navigation of autonomous vehicles.
  - **real-time optimization** can effectively manage time-varying constraints providing **safety guarantees**.
- Ideal solution for applications that require **high-level decision-making.**

Survey on the Impact of Advanced Control – IFAC's Industry Committee [6]	
Current Impact	**Future Impact**
1. PID	1. Model-predictive control
2. System Identification	2. PID
3. Estimation and Filtering	3. Fault Detection and Identification
4. Model-predictive control	4. System Identification
5. Fault Detection and Identification	5. Process data analytics
6. Process data analytics	6. Estimation and Filtering
7. Decentralized and/or coordinated control	7. Decentralized and/or coordinated control
8. Robust control	8. Intelligent control
9. Intelligent control	9. Adaptive control
10. Adaptive control	10. Robust control
11. Nonlinear control	11. Nonlinear control
12. Discrete-event systems	12. Discrete-event systems
13. Other advanced control technologies	13. Hybrid dynamical systems
14. Hybrid dynamical systems	14. Other advanced control technologies
15. Repetitive control	15. Repetitive control
16. Game theory	16. Game theory

MPC is likely to play an increasingly important role in shaping the future of transportation.

[6] Samad, Tariq, et al. "Industry engagement with control research: Perspective and messages." *Annual Reviews in Control* 49 (2020): 1-14.

End of slides

Model Predictive Control for constrained navigation of autonomous vehicles

[1] **Saccani** D., et al. Model predictive control for multi-agent systems under limited communication and time-varying network topology. In: 2023 IEEE Conference on Decision and Control (CDC).
[2] **Saccani**, D., & Fagiano, L. Autonomous uav navigation in an unknown environment via multi-trajectory model predictive control. In 2021 European Control Conference (ECC)
[3] **Saccani**, D., Cecchin, L., & Fagiano, L. Multitrajectory model predictive control for safe UAV navigation in an unknown environment. IEEE Transactions on Control Systems Technology
[4] Cecchin, L., **Saccani**, D., & Fagiano, L. G-beam: Graph-based exploration and mapping for autonomous vehicles. In 2021 IEEE Conference on Control Technology and Applications (CCTA)
[5] Bolognini, M., **Saccani**, D., Cirillo, F., & Fagiano, L. Autonomous navigation of interconnected tethered drones in a partially known environment with obstacles. In 2022 IEEE CDC.