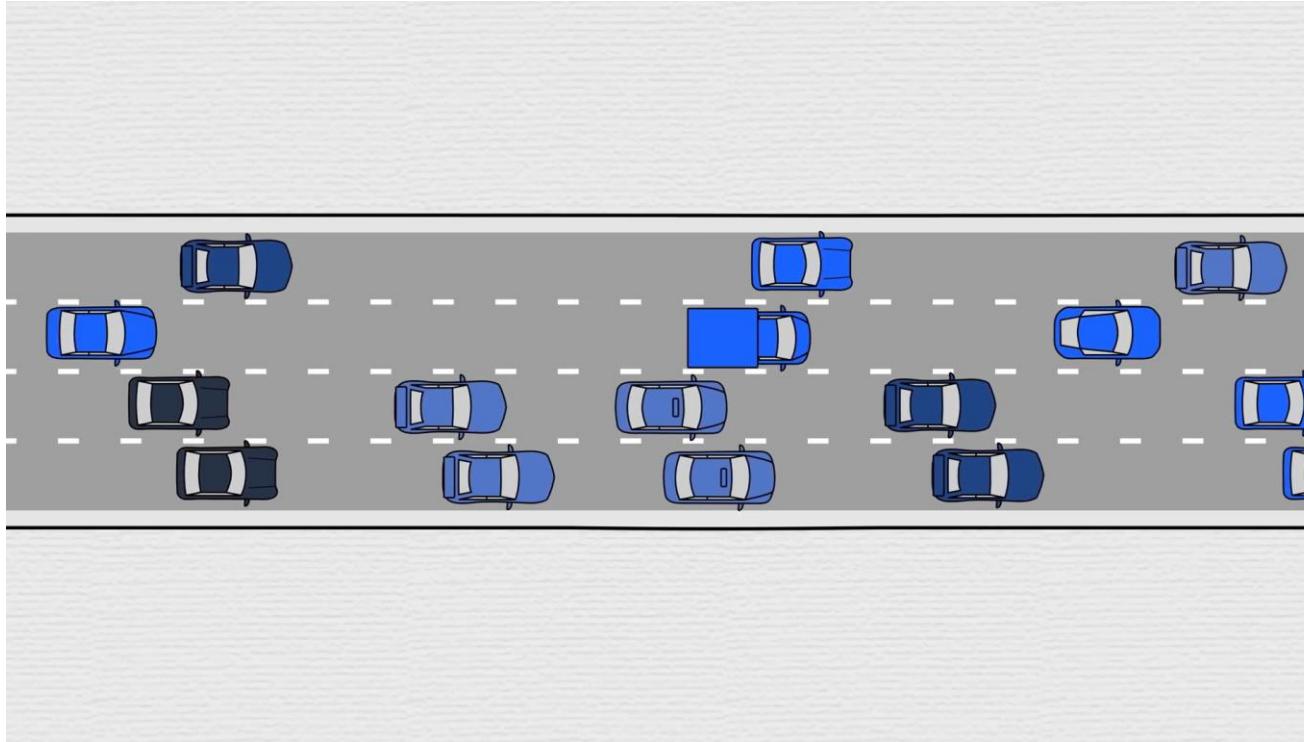


# Macroscopic Modelling and Control of Mixed Traffic



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# Generalization of the problem

- Traffic flow dynamics
- Macroscopic models
- Connected and Autonomous Vehicles (CAVs).



- Existing models and challenges.

# Objectives

The objectives of this study are to formulate Second Order Traffic Model (SOTFM) for Mixed-Traffic, can be used to:

- Describe the dynamics of Mixed-Traffic in the presence of CAVs and Human-Driven Vehicles (HDVs).

Consequently:

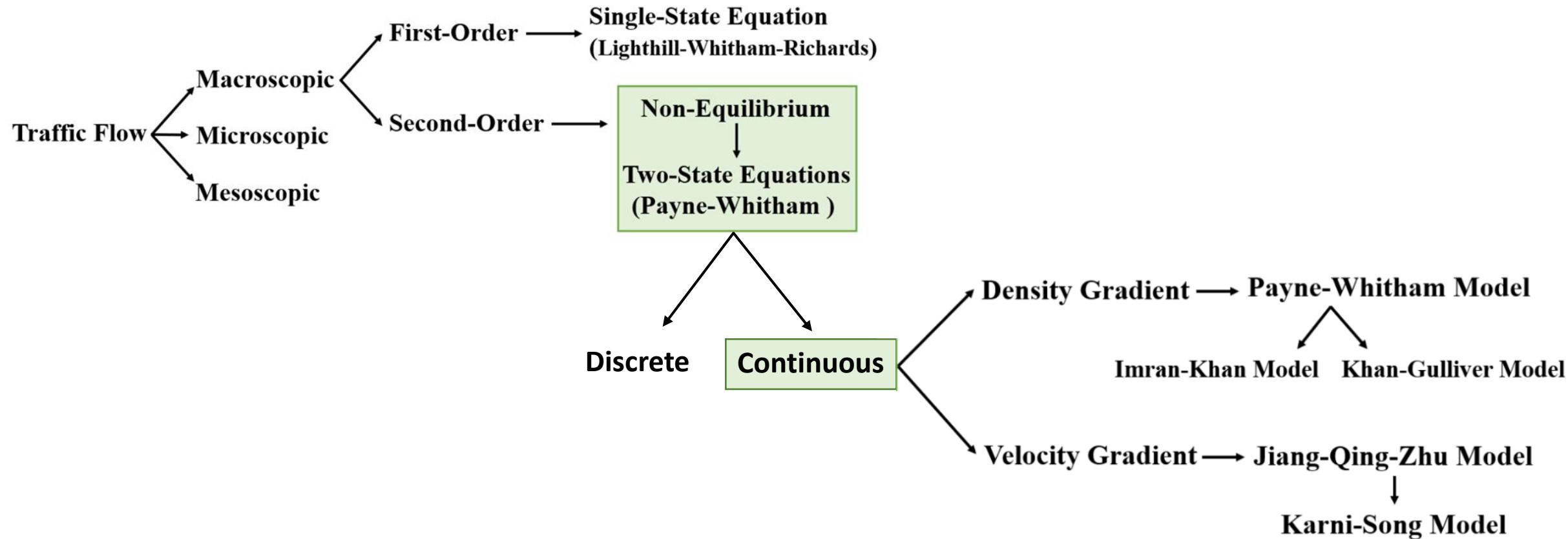
- Analysis of traffic flow.
- Traffic control applications.

## **Understanding of the existing macroscopic models**

### **Aims:**

- Expansion of model tree
- Performance assessment
- Computational efficiency

## Model Tree



- **Density gradient:** Presumption of forward condition is based on spatial changes in density
- **Velocity gradient:** Presumption of forward condition is based on spatial changes in velocity

# The second-order macroscopic models

- $$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial(v(x,t) \cdot \rho(x,t))}{\partial x} = 0, \quad \dots (1)$$
- $$\frac{\partial v(x,t)}{\partial t} + (v(x,t) - c_0) \frac{\partial v(x,t)}{\partial t} = \frac{V(\rho(x,t)) - v(x,t)}{T}. \quad \dots (2)$$

- JQZ model
- Uniform characterization

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$v(x, t)$ : velocity;  $\rho(x, t)$ : density;  $c_0$ : velocity constant;  $V(\rho(x, t))$ : Velocity-Density relationship;  $T$ : Relaxation time;

**JQZ:** Jiang, Qing and Zhu.

- **Ground truth:** Simulation of Urban Mobility (SUMO)
- **Study Area:** A 20,000 m highway with closed boundary conditions
- **Initial traffic conditions (SUMO and SOTFMs):**

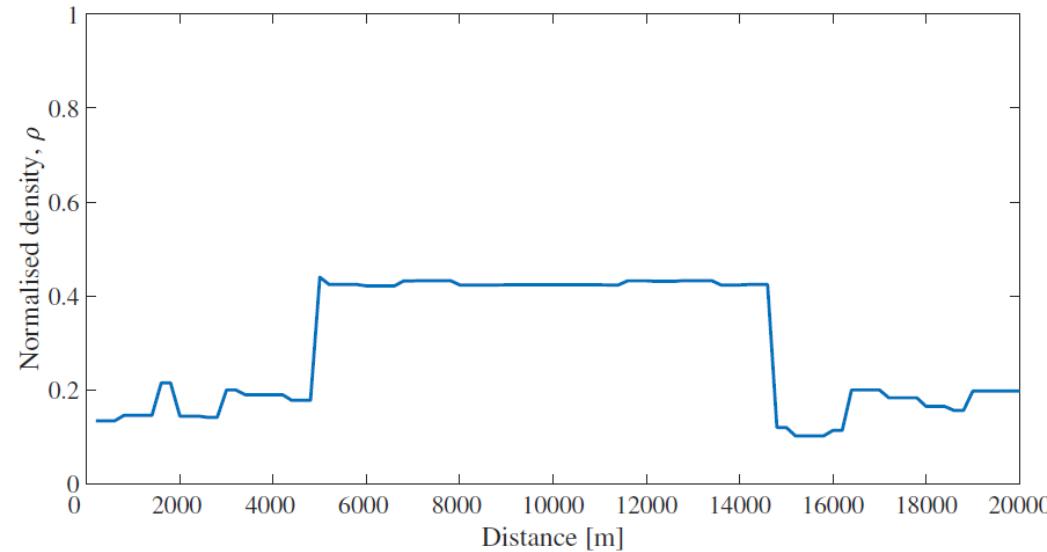


Figure 1: Initial density distribution – scenario I

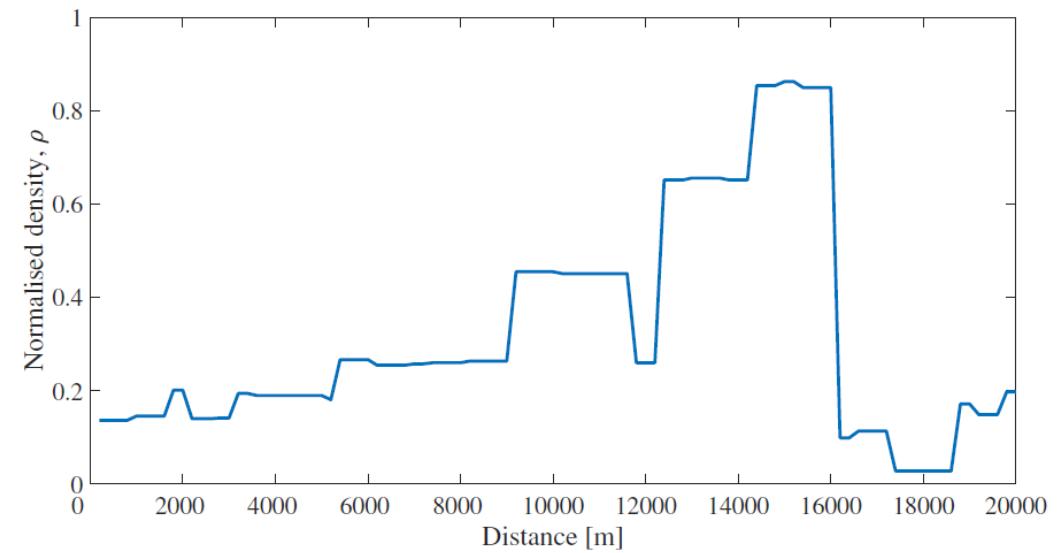


Figure 2: Initial density distribution - scenario II

- **Discretization methods for SOTFMs:**
  - Density gradient models: First-Order Centered (FORCE) scheme
  - Velocity gradient models: Finite difference (conserved) scheme

# Performance Assessment

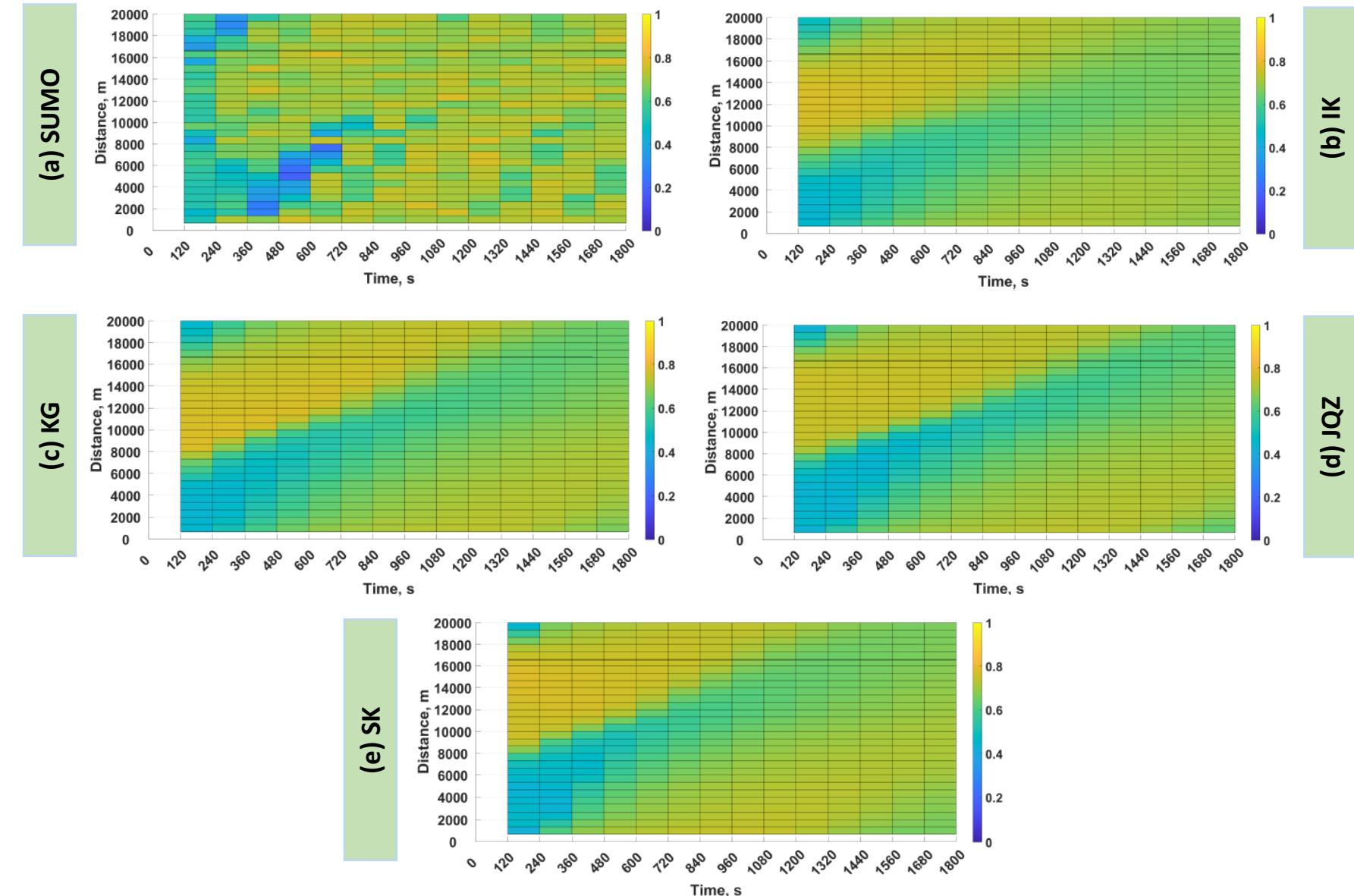


Figure 3: Spatiotemporal flow analysis (SUMO vs SOTFMs) – scenario I.

# Performance Assessment

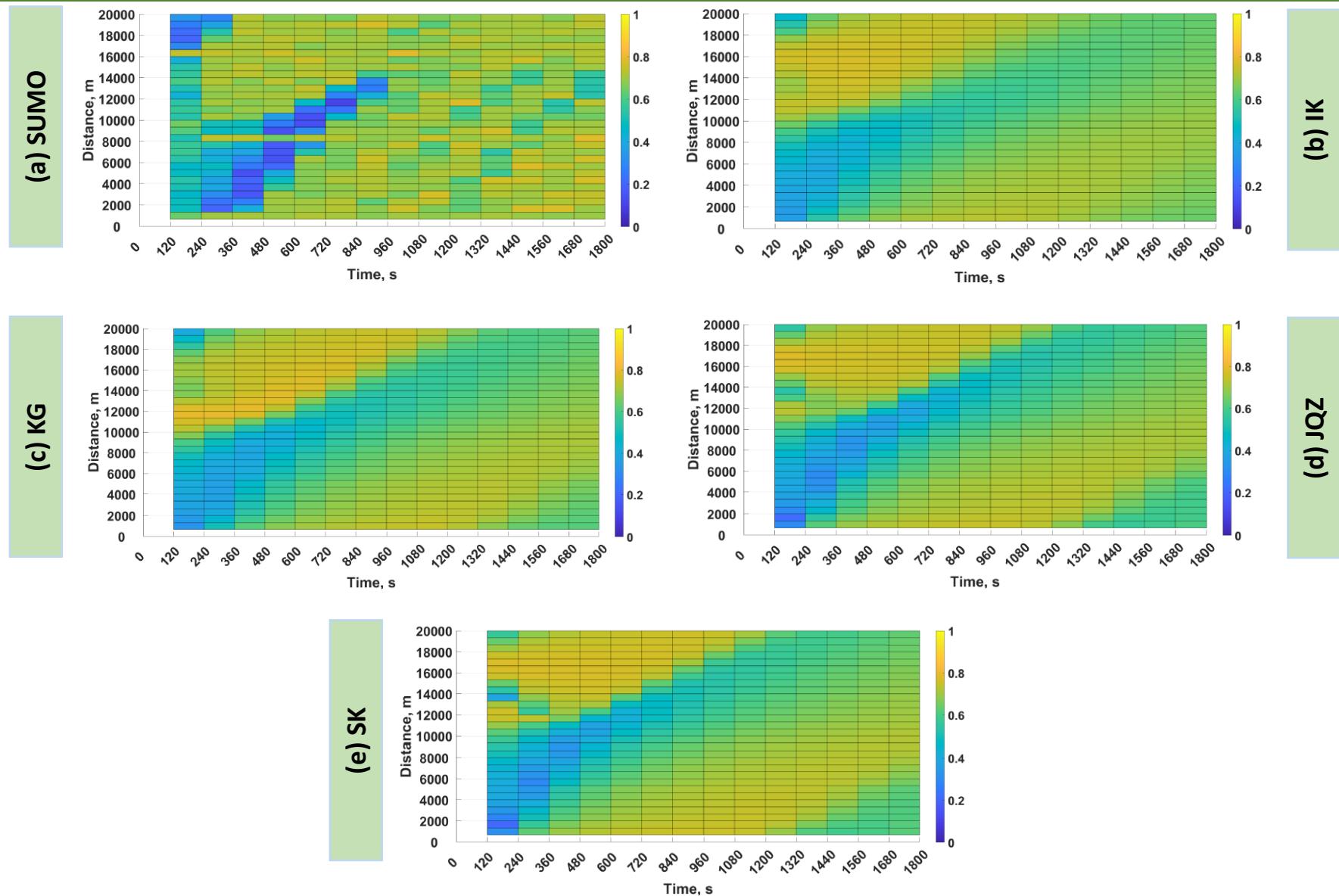


Figure 4: Spatiotemporal flow analysis (SUMO vs SOTFMs) – Scenario II.

Density gradient

Velocity gradient

**Table 1:** Summary

Model	PI (I/II)	Parameter	Admissible Range	Calibrated Value (I/II)	Unit
IK	0.078 / 0.10	$b$	0.1 – 1	0.75 / 0.95	[]
		$v_f$	28 - 32	30.72 / 30.78	m/s
		$\tau$	5 – 20	5.28 / 19.62	s
		$\tau_h$	1 – 20	17.23 / 13.87	s
KG	0.083 / 0.10	$\tau_r$	0.7 – 1.7	0.78 / 0.73	s
		$v_f$	28 - 32	30.70 / 30.68	m/s
		$\tau$	5 – 20	5.12 / 19.42	s
		$s_{st}$	1 – 5	3.15 / 4.42	m
JQZ	0.091 / 0.125	$c_0$	5 - 25	24.81 / 15.56	m/s
		$v_f$	28 – 32	30.09 / 30.21	m/s
		$\tau$	5 – 20	19.65 / 19.95	s
SK	0.089 / 0.12	$s_{min}$	1 – 6	5.51 / 4.51	m
		$v_f$	28 – 32	30.09 / 30.27	m/s
		$\tau$	5 – 20	19.66 / 19.48	s
		$\tau_r$	0.7 – 1.7	0.78/0.99	s
		$L_c$	7 – 10	9.18 / 7.40	m

**PI:** Performance Index; **IK:** Imran and Khan; **KG:** Khan and Gulliver; **JQZ:** Jiang, Qing and Zhu; **SK:** Song and Karni

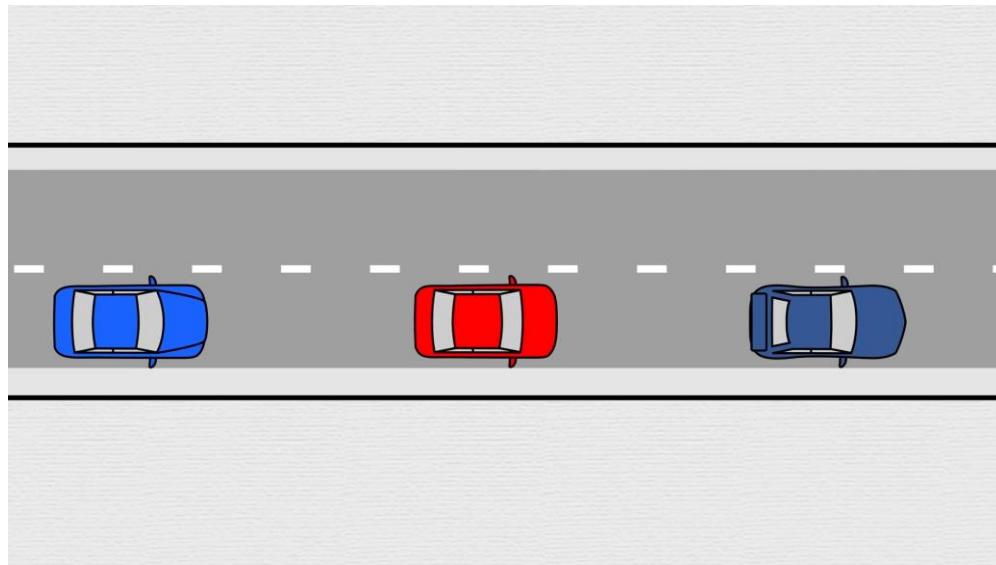
**Table 2:** Computational efficiency

Model	Average Simulation Time [s] (I/II)	Std [s] (I/II)	Coefficient of Variation (I/II)
IK	0.4376 / 0.5773	0.1553 / 0.2732	0.3548 / 0.3548
KG	0.3998 / 0.4121	0.0712 / 0.0744	0.1780 / 0.1780
JQZ	0.0013 / 0.0013	$2.70 \cdot 10^{-4}$ / $2.76 \cdot 10^{-4}$	0.2125 / 0.2125
SK	0.0014 / 0.0015	$3.08 \cdot 10^{-4}$ / $3.17 \cdot 10^{-4}$	0.2126 / 0.2126

# Characterization of Mixed Traffic

## Aims:

- New model for mixed traffic.
- Calibration and validation
- Practical utility in traffic analysis
- Application in traffic control



$$H_a = L + \frac{v(x,t)^2}{2d_m(HDV)} + (v(x,t) \cdot (\tau_r + \tau_h)_{HDV}) \dots (3)$$

$$\zeta_* = (1-\sigma) \frac{H_a}{\tau_a}, \dots (4)$$

$$H_b = L + \frac{v(x,t)^2}{2d_m(HDV)} + (v(x,t) \cdot (\tau_r + \tau_h)_{CAV}) \dots (5)$$

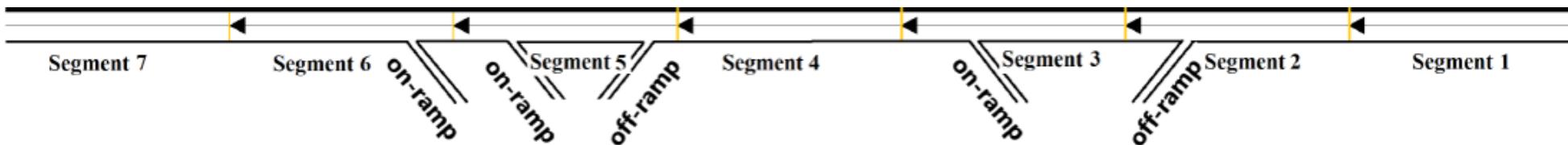
$$\varphi_* = (\sigma) \frac{H_b}{\tau_b}, \dots (6)$$

$$\left\{ \begin{array}{l} \frac{\partial \rho(x,t)}{\partial t} + \frac{\partial(v(x,t) \cdot \rho(x,t))}{\partial x} = 0, \\ \frac{\partial v(x,t)}{\partial t} + (v(x,t) - [\zeta_* + \varphi_*]) \frac{\partial v(x,t)}{\partial x} = \frac{V(\rho(x,t)) - v(x,t)}{T}. \end{array} \right. \dots (7)$$

**Table 3:** Model parameters

$v$	Velocity
$\rho$	Density
$V(\rho(x,t))$	Equilibrium Velocity
$T$	Relaxation Time of each class
$H_a$	Headway of HDVs
$H_b$	Headway of CAVs
$\zeta_*$	Reaction Velocity of HDVs
$\varphi_*$	Reaction Velocity of CAVs
$\tau_r$	Reaction time
$\tau_h$	Harmonization time
$d_m$	Maximum deceleration

- **Ground truth:** SUMO
- **Study area:** Dublin motorway (M-50), IE [1].
- **Data:** Historical traffic data (Transport infrastructure Ireland) based on conventional traffic [2].
- **Corridor length:** 11 kms



**Figure 5:** Representation of the motorway corridor

$$\rho_i^{n+1} = \rho_i^n + \frac{\Delta t}{\Delta x} (\rho_i^n (v_i^n - v_{i+1}^n)) + \frac{\Delta t}{\Delta x} (v_i^n (\rho_{i-1}^n)), \dots \quad (8)$$

- **Discretized model:**  $\left\{ \begin{array}{l} \text{if } v_i^n \leq [\zeta_* + \varphi_*]: v_i^{n+1} = v_i^n + \frac{\Delta t}{\Delta x} ([\zeta_* + \varphi_*] - v_i^n) (v_{i+1}^n - v_i^n) + \frac{\Delta t}{T} (V(\rho_i^n) - v_i^n), \dots \quad (9) \\ \text{else: } v_i^{n+1} = v_i^n + \frac{\Delta t}{\Delta x} ([\zeta_* + \varphi_*] - v_i^n) (v_i^n - v_{i-1}^n) + \frac{\Delta t}{T} (V(\rho_i^n) - v_i^n). \dots \quad (10) \end{array} \right.$

[1] [https://github.com/maxime-gueriau/ITSC2020\\_CAV\\_impact](https://github.com/maxime-gueriau/ITSC2020_CAV_impact)

[2] <https://traffic.tii.ie/>

$$v(\rho(x, t)) = v_f \cdot \exp\left[\frac{-1}{c} \left(\frac{\rho(x, t)}{\rho_{cr}}\right)^c\right] \quad \dots (11)$$

$v_f$ : free flow velocity;  $\rho_{cr}$ : critical density;  $c$ : shape factor

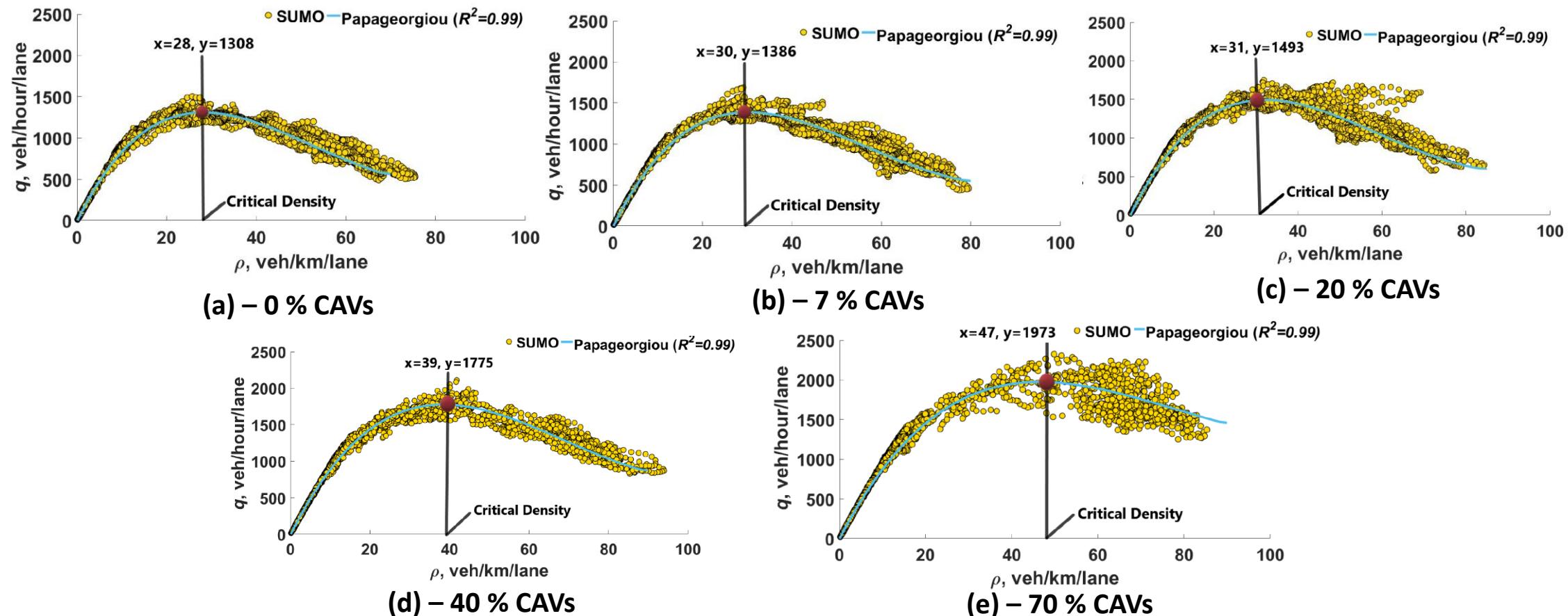


Figure 5: SUMO vs Papageorgiou model.

Papageorgiou, M., Blosseville, J.-M., & Hadj-Salem, H. (1989). Macroscopic modelling of traffic flow on the Boulevard Périphérique in Paris. *Transportation Research Part B: Methodological*, 23(1), 29–47. [https://doi.org/10.1016/0191-2615\(89\)90021-0](https://doi.org/10.1016/0191-2615(89)90021-0)

**Table 3:** Comparison of the proposed SOTFM and the JQZ model.

PR (%) of CAVs	RMSPE of the proposed model		RMSPE of the JQZ model	
	Density	Velocity	Density	Velocity
0	1.24	1.80	3.75	5.33
7	1.22	1.69	3.71	5.24
20	1.19	1.60	3.65	5.21
40	1.12	1.35	3.43	5.05
70	0.91	1.07	1.90	3.01

**RMSPE:** Root Mean Square Percentage Error; **PR:** Penetration Rate

## **Utility in Traffic Analysis**

# Practical utility of the proposed model

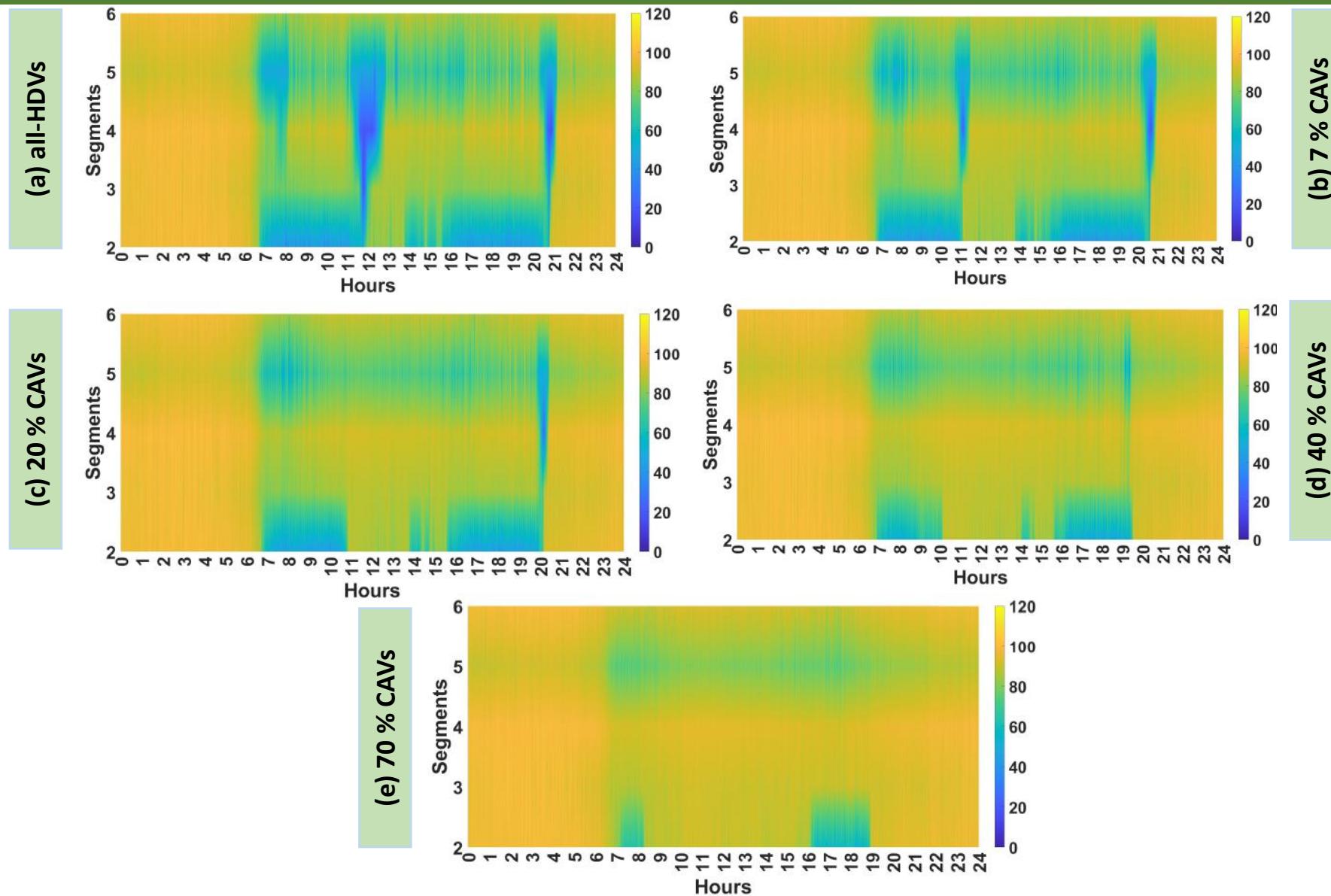
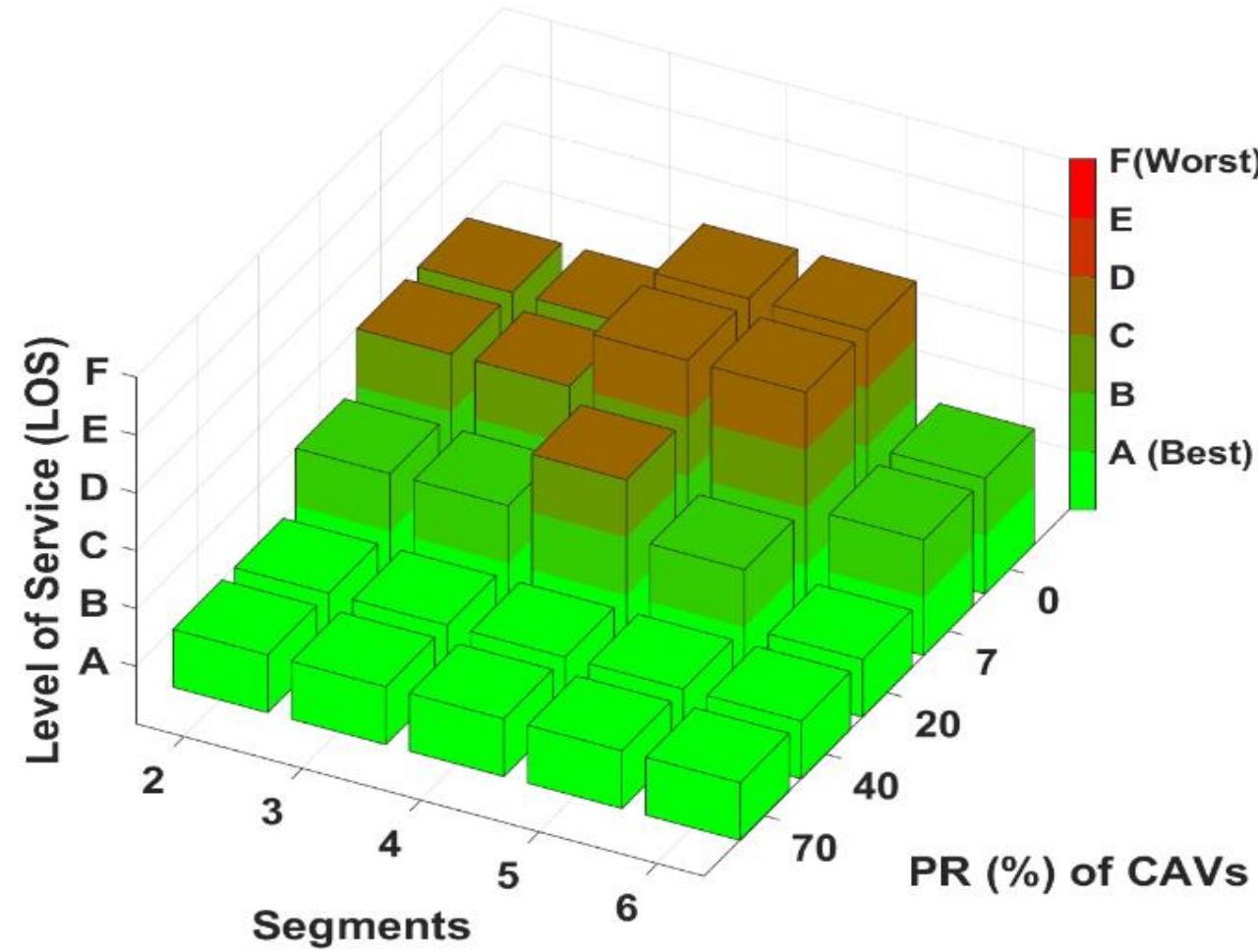


Figure 6: Spatiotemporal velocity evolution over the Dublin motorway corridor.

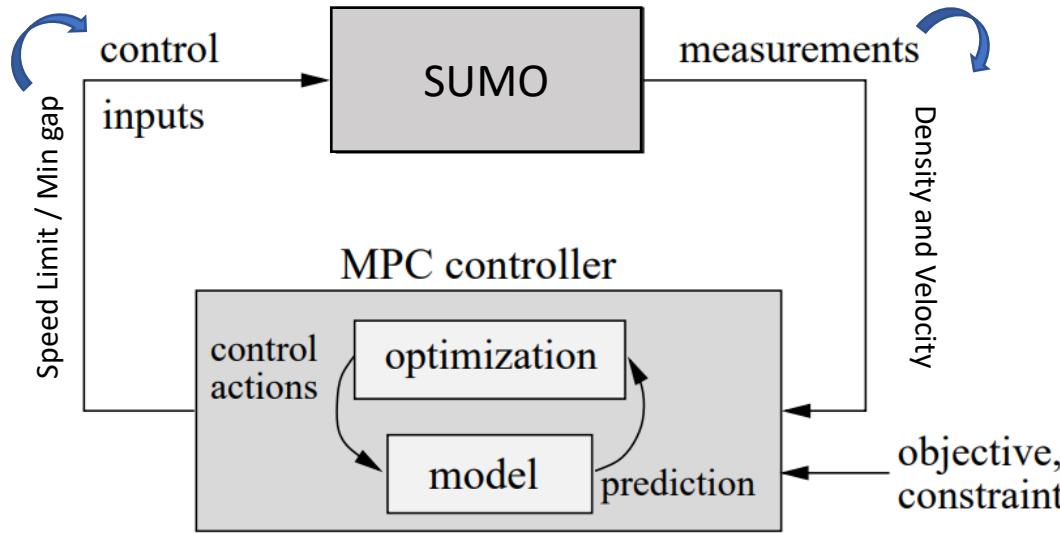
Ph.D. Award Presentation

IEEE Intelligent Transportation Systems Society – Italian Chapter



**Figure 7:** Level of Service (LOS) demonstration over the Dublin motorway during the evening peak hour evaluated with the proposed SOTFM.

## **Applications in Traffic Control**



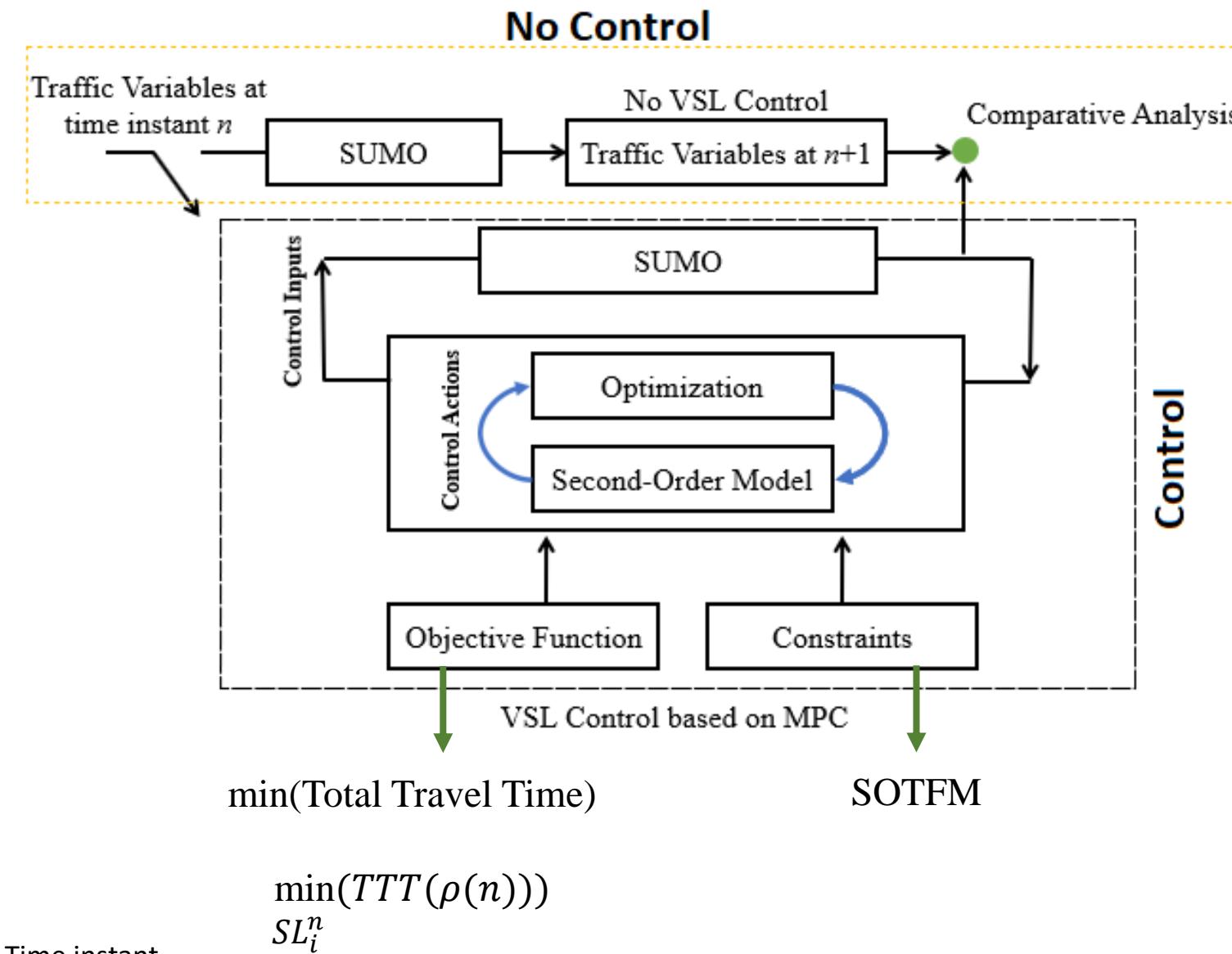
**Figure 8:** MPC framework.

- SUMO (TraCi4MATLAB, MATLAB)
- Prediction Model (SOTFM)

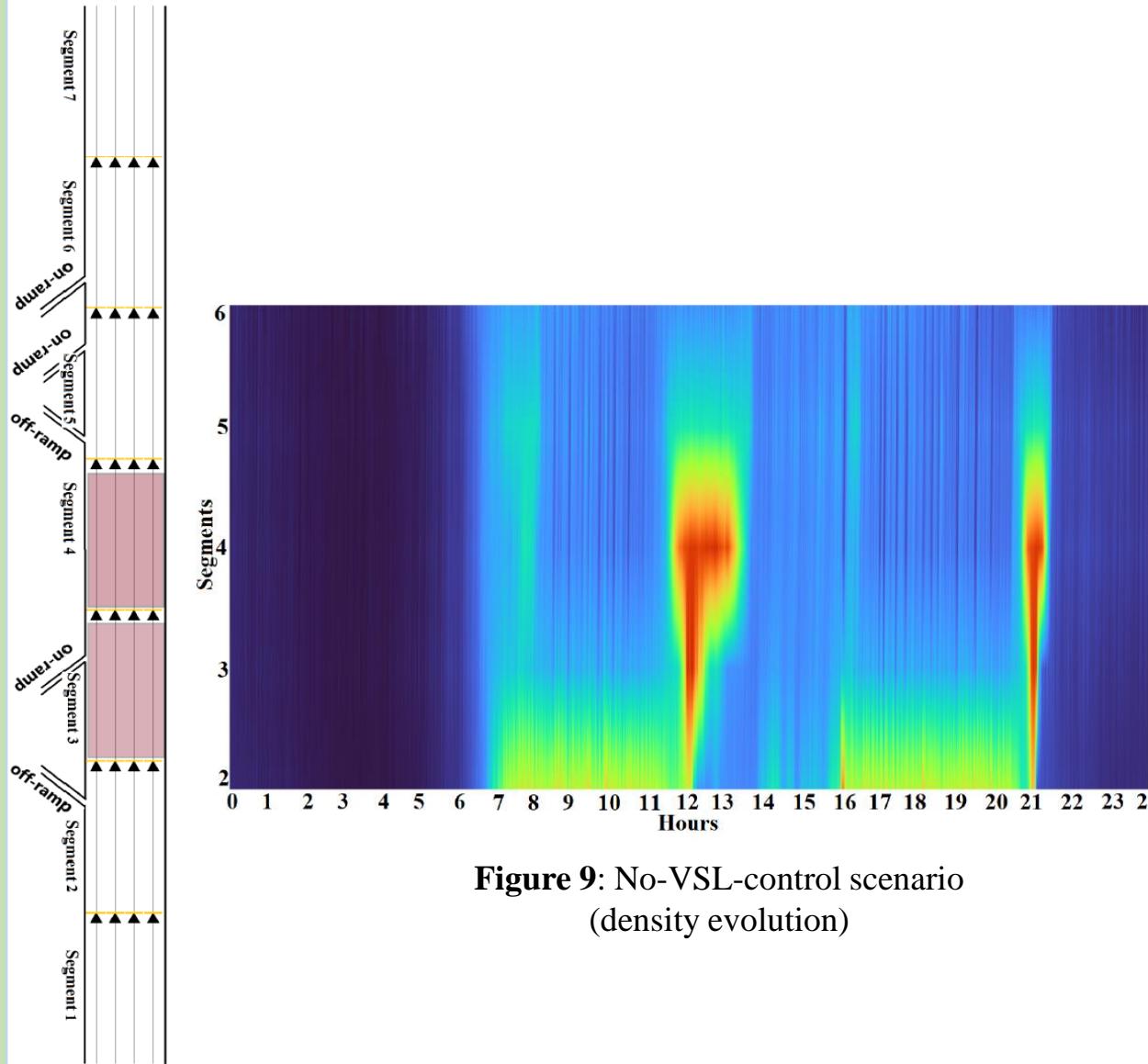
NEXT?

- Conventional Control with Variable Speed Limit (VSL)
- A Novel Regulation of Motorway Traffic with CAVs

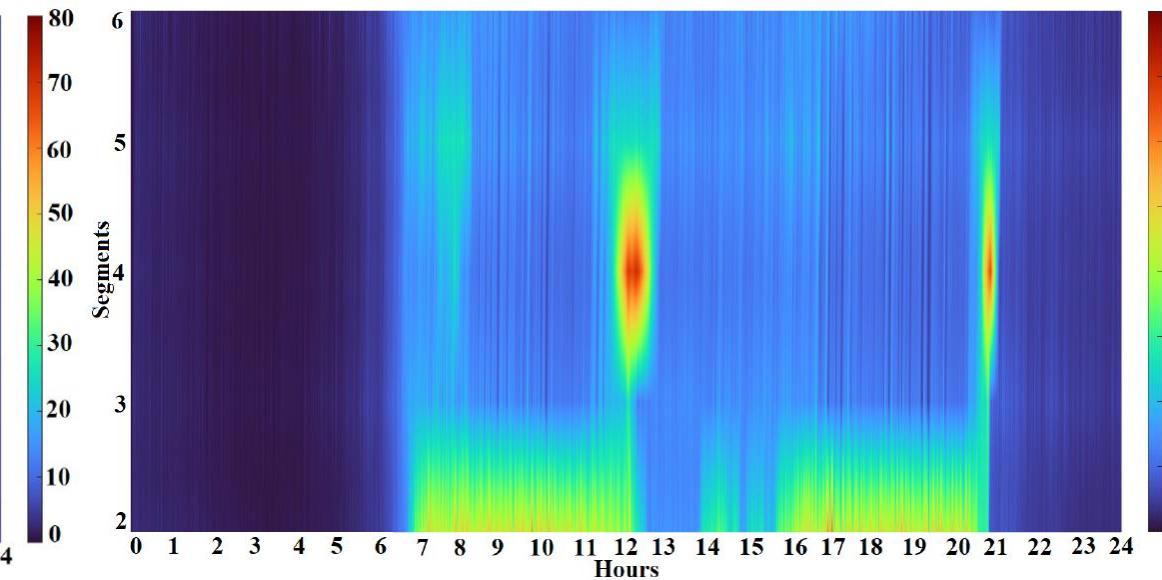
# Conventional Control with VSL employing a Continuum Model as a Prediction Model



# MPC Driven VSL Control of Traffic

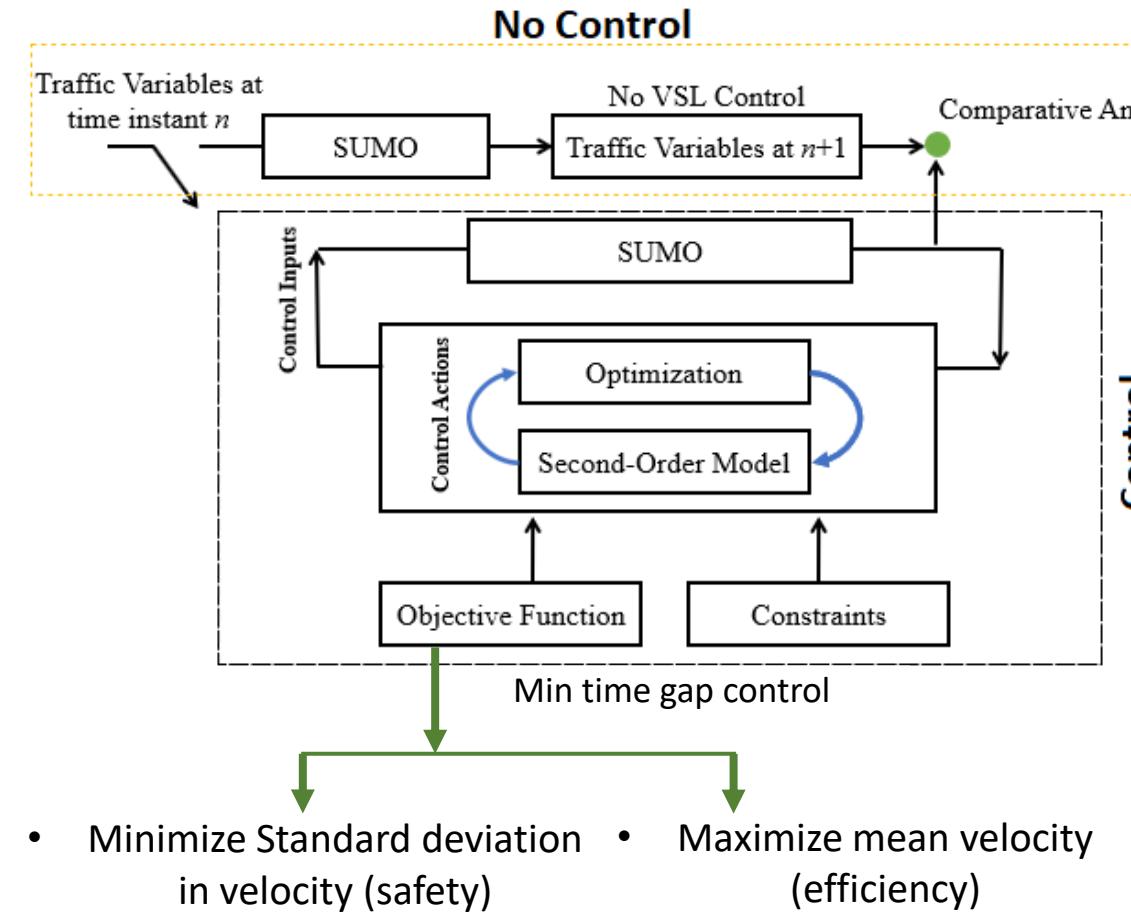


**Figure 9:** No-VSL-control scenario  
(density evolution)



**Figure 10:** VSL-control scenario  
(density evolution)

# A Novel regulation of Motorway Traffic with CAVs (CAVs penetration rate 40 %)



$$\theta = (\theta_1 - \theta_2) : \min_{\tau}(\theta)$$

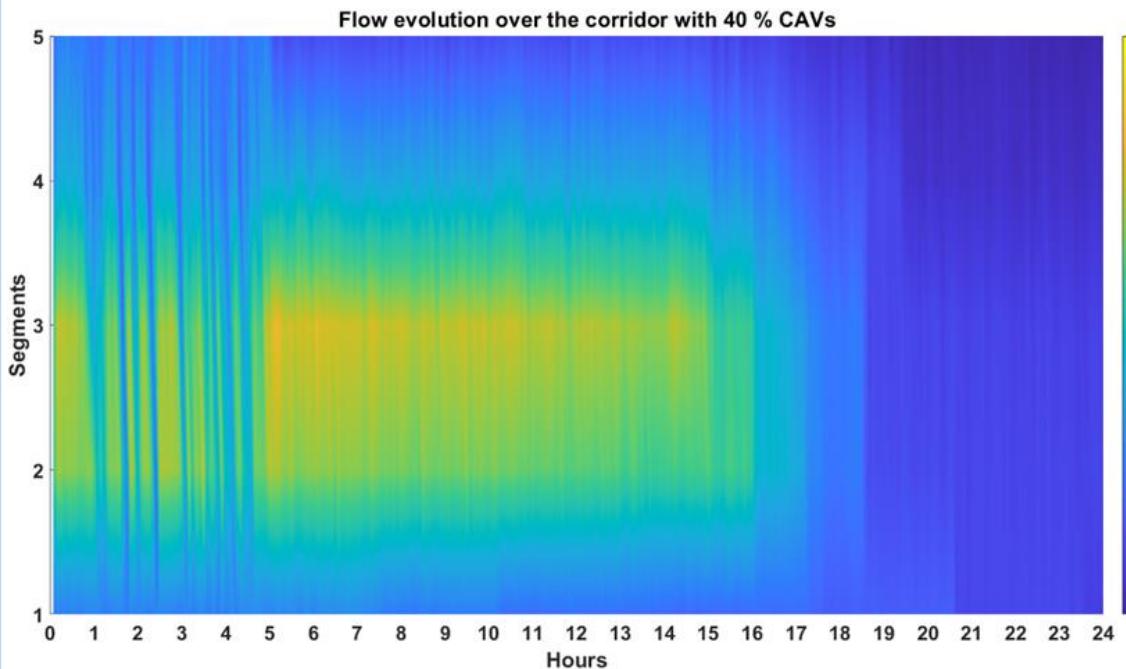
$$\theta_1 = \sum_i \frac{v_i(\tau) - \bar{v}_i}{N-1}, \text{ minimize}$$

$$\theta_2 = \sum_i \frac{v_i(\tau)}{N}, \text{ maximize}$$

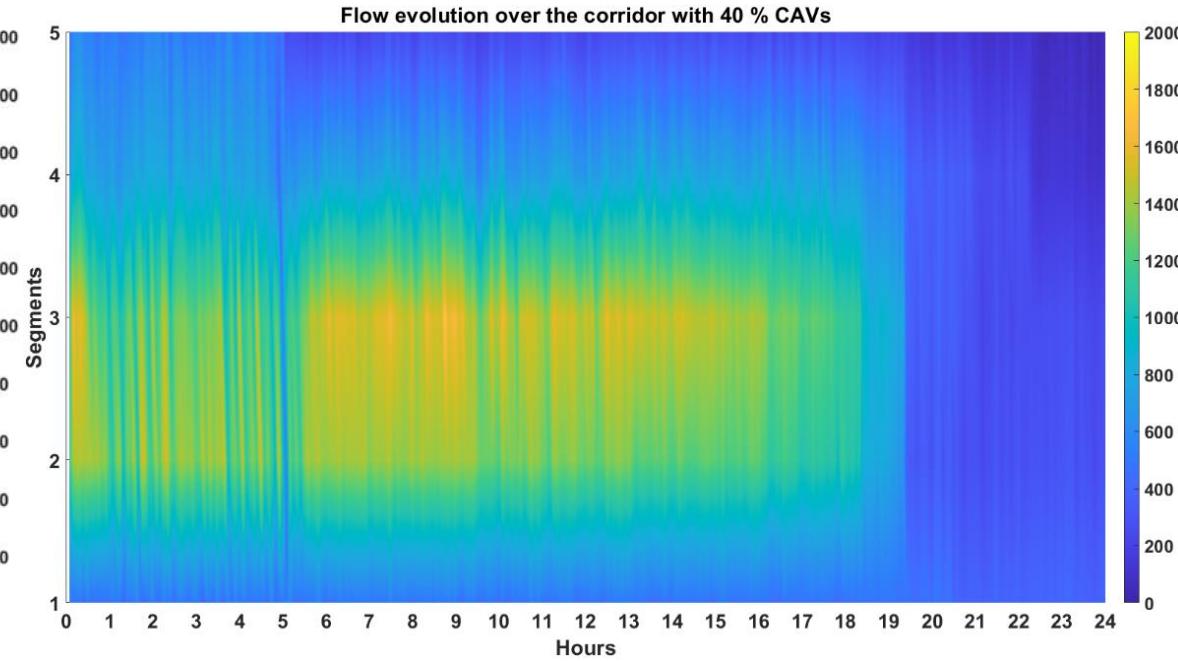
$$\theta = \frac{\theta_1}{\theta_{1,max}} - \frac{\theta_2}{\theta_{2,max}}$$

$\bar{v}_i$ : mean velocity (segment)

# A Novel regulation of Motorway Traffic with CAVs (CAVs penetration rate 40 %)



**Figure 12:** No-control scenario  
(flow evolution)



**Figure 13:** Control scenario  
(flow evolution)

- ✓ Characterization of self-organizing behavior of traffic flow with CAVs
- ✓ Traffic flow and emissions profiling driven on the fundamental diagram of mixed traffic

## Outcomes

Research Articles			
Title	Status	Journal	
1. Macroscopic evaluation of traffic flow in view of connected and autonomous vehicles: A simulation-based approach	Published	Alexandria Engineering Journal	
2. Macroscopic modeling of connected, autonomous and human-driven vehicles: A pragmatic perspective	Published	Transportation Research Interdisciplinary Perspectives	
3. Appraisal of the second-order traffic systems employing sensitivity analysis and numerical simulation	Submitted	Transportation Research Interdisciplinary Perspectives	
4. Traffic flow and emissions profiling driven on the fundamental diagram of mixed traffic	Ready	To be decided	
5. An MPC-driven VSL control of motorway traffic	Submitted	Transportation Engineering	
6. A novel MPC-driven regulation of traffic flow with connected and autonomous vehicles	Under-prep	To be decided	

## Recommendations

- Characterization of mixed traffic employing other macroscopic models, such as METANET and Cell Transmission Model (CTM), others.
- Practical utilization of the proposed models in weather based adaptive control and pavement conditions based adaptive control.
- Use of the proposed model in Hybrid simulation approach
- Safety evaluation of traffic employing surrogate measures in the framework of SOTFMs.
- Business modules, product development for vehicles based on macroscopic systems for advanced decision making.
- Comparison of conventional control strategies with the proposed novel control strategy in a more complex traffic scenarios.

1. **W. Imran**, T. Tettamanti, B. Varga, G. N. Bifulco, and L. Pariota, “Macroscopic modeling of connected, autonomous and human-driven vehicles: A pragmatic perspective,” *Transp. Res. Interdiscip. Perspect.*, vol. 24, p. 101058, Mar. 2024
2. **W. Imran** and L. Pariota, “Macroscopic evaluation of traffic flow in view of connected and autonomous vehicles: A simulation-based approach,” *Alexandria Engineering Journal*, vol. 79, pp. 581–590, 2023.
3. **W. Imran**, Z. H. Khan, T. A. Gulliver, M. Alam, and K. S. Khattak, “Non-homogeneous traffic characterization based on driver reaction and stimuli,” *Transportation Research Interdisciplinary Perspectives*, vol. 21, p. 100858, 2023.
4. Z. H. Khan, **W. Imran** et al., “Macroscopic traffic characterization based on driver memory and traffic stimuli,” *Transportation Engineering*, vol. 14, p. 100208, 2023.
5. Z. H. Khan, T. A. Gulliver, **W. Imran**, K. S. Khattak, A. B. Altamimi, and A. Qazi, “A macroscopic traffic model based on relaxation time,” *Alexandria Engineering Journal*, vol. 61, no. 1, pp. 585–596, 2021.

## General:

1. Z. H. Khan, A. B. Altamimi, **W. Imran**, M. Alsaffar, K. S. Khattak, and F. F. Alfaisal, “Macroscopic Traffic Modelling on the Impact of Road Surface Potholes: Development and Numerical Solution,” *IEEE Access*, vol. 12, pp. 81718–81735, 2024
2. D. Khan, Z.H. Khan, **W. Imran**, K.S. Khattak, T. A. Gulliver, “Macroscopic Flow Characterization at T-Junctions”, *Transportation Research Interdisciplinary Perspectives*, vol. 14, 2022.
3. S. Islam, Z. H. Khan, T. A. Gulliver, K. S. Khattak, and **W. Imran**, “Pedestrian traffic characterization based on pedestrian response,” *IEEE Access*, vol. 10, pp. 118397–118408, 2022.
4. **W. Imran**, Z. H. Khan, T. A. Gulliver, K. S. Khattak, S. Saeed, and M. S. Aslam, “Macroscopic Traffic Flow Characterization for Stimuli Based on Driver Reaction,” *Civil Engineering Journal*, vol. 7, no. 1, pp. 1–13, 2021.
5. Z. H. Khan, T. A. Gulliver, and **W. Imran**, “A Macroscopic Traffic Model Based on the Safe Velocity at Transitions,” *Civil Engineering Journal*, vol. 7, no. 6, pp. 1060–1069, 2021.
6. A. Khan, K. Khattak, Z. Khan, T. Gulliver, **W. Imran**, and N. Minallah, “Internet-of-Video Things Based Real-Time Traffic Flow Characterization,” *ICST Transactions on Scalable Information Systems*, vol. 8, no. 33, p. 171596, 2021.
7. M. Sarir, R. Khan, M. Alam, M. T. Khan, and **W. Imran**, “Performance Evaluation of Asphalt Concrete Mixtures Using Bagasse Ash as Filler,” *Iranian Journal of Science and Technology, Transactions of Civil Engineering*, 2021.
8. **W. Imran**, Z. H. Khan, T. A. Gulliver, K. S. Khattak, and H. Nasir, “A macroscopic traffic model for heterogeneous flow,” *Chinese Journal of Physics*, vol. 63, pp. 419–435, 2020.
9. Z. H. Khan, **W. Imran**, T. A. Gulliver, K. S. Khattak, Z. Wadud, and A. N. Khan, “An Anisotropic Traffic Model Based on Driver Interaction,” *IEEE Access*, vol. 8, pp. 66799–66812, 2020.
10. Z. H. Khan, **W. Imran**, S. Azeem, K. S. Khattak, T. A. Gulliver, and M. S. Aslam, “A Macroscopic Traffic Model based on Driver Reaction and Traffic Stimuli,” *Applied Sciences*, vol. 9, no. 14, p. 2848, 2019.

# Q & A

# Thank You!

## **Characterization of Self-Organizing Behavior of Traffic Flow with CAVs**

### **Aims:**

- Modeling the anticipative behavior of CAVs
- Spatiotemporal analysis of traffic variables

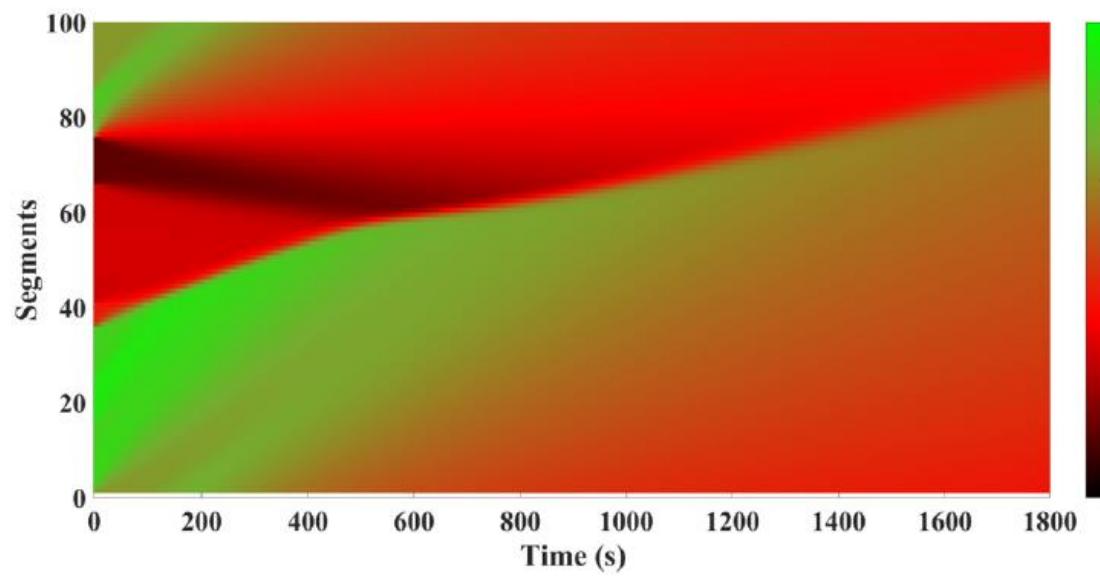
$$\left\{ \begin{array}{l} \frac{\partial \rho}{\partial t} + \frac{\partial(v(x,t) \cdot \rho(x,t))}{\partial x} = 0, \\ \frac{\partial v(x,t)}{\partial t} + \left( v(x,t) - \frac{l_g + \tau_r(\rho) \cdot v(x,t)}{\tau_r(\rho)} \right) \frac{\partial v(x,t)}{\partial x} = \frac{V(\rho(x,t)) - v(x,t)}{T}. \end{array} \right. \dots (15)$$

$$\tau_r(\rho) = \frac{\tau_{r(min)}}{\frac{\rho(x,t)}{\rho_m}}$$

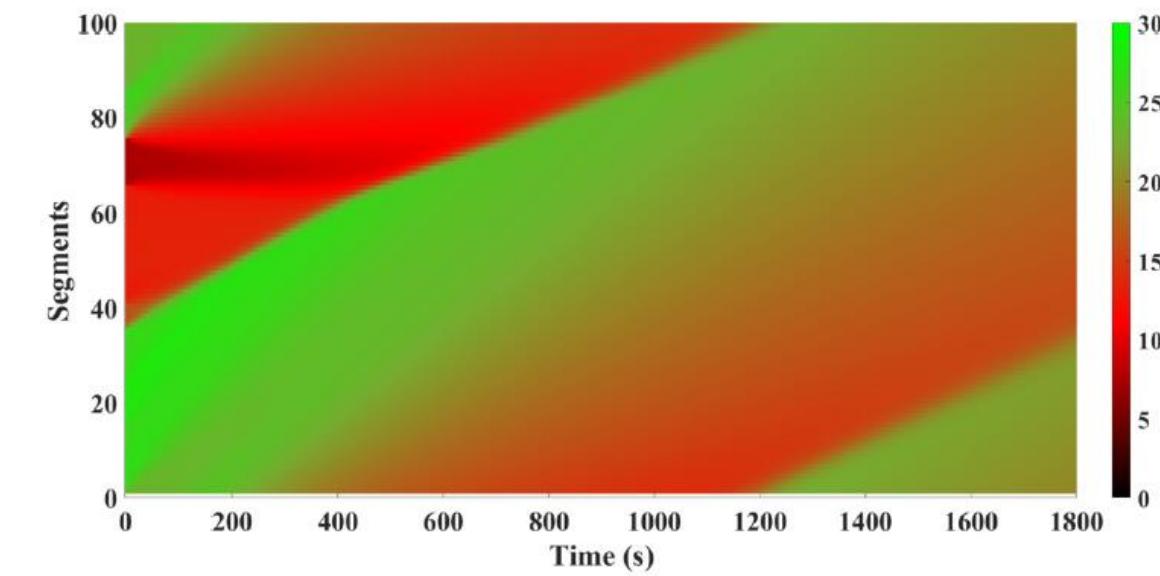
**Table 4:** Parameters

Parameters	Description
$\tau_r(\rho)$	Reaction time as function of density
$\tau_{r(min)}$	Min reaction time
$l_g$	Lengthways gap

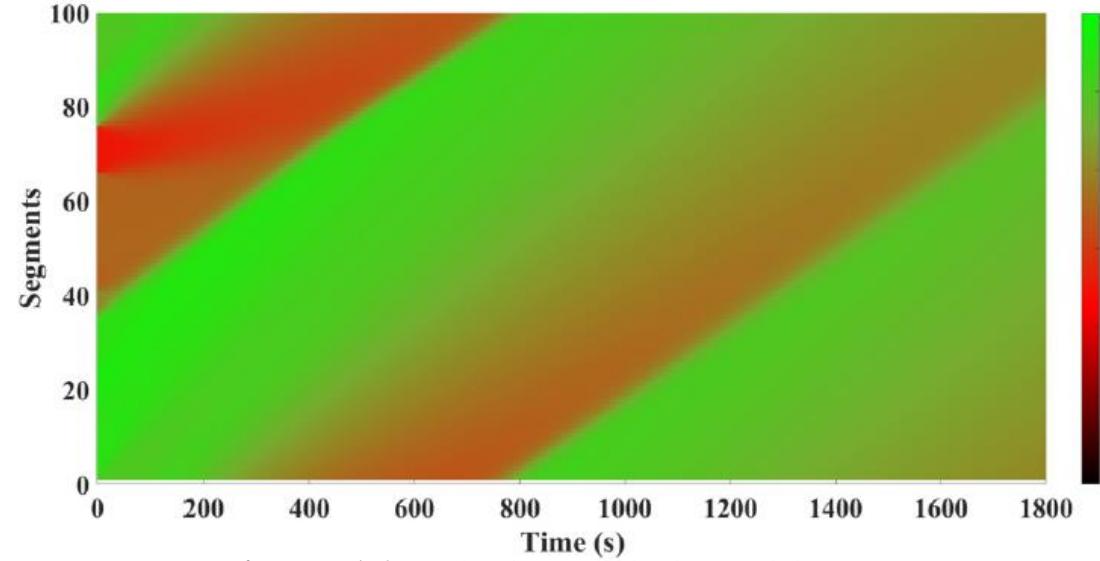
\* W. Imran and L. Pariota, "Macroscopic evaluation of traffic flow in view of connected and autonomous vehicles: A simulation-based approach," Alexandria Engineering Journal, vol. 79, pp. 581–590, 2023.



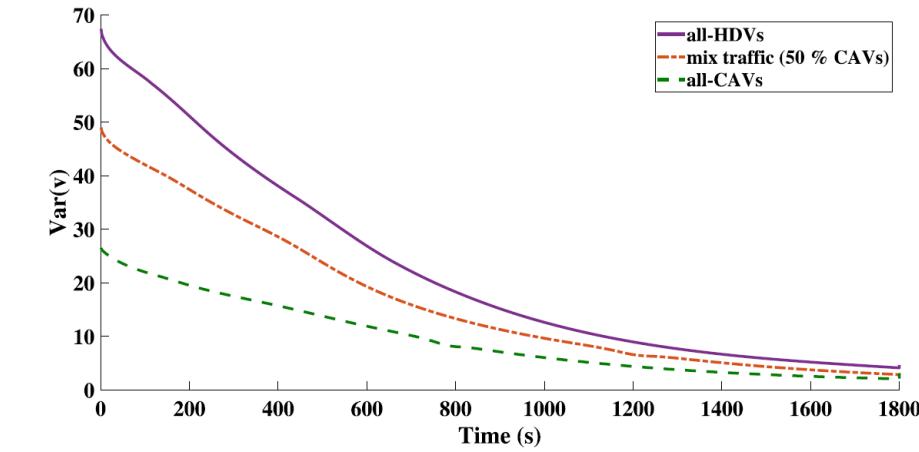
**Figure 14:** Velocity evolution with HDVs



**Figure 15:** Velocity evolution with CAVs and HDVs (50 % each)



**Figure 16:** Velocity evolution with CAVs



**Figure 17:** Variance of velocity